Elliptical Galaxies

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● Basic properties of elliptical galaxies

● Formation of elliptical galaxies
Photometric Properties

Isophotes of elliptical galaxies are usually fitted by ellipses:

Major axis $a$; minor axis $b$, ellipticity $\varepsilon = 1 - b/a$.

Types of ellipticals:

$$E_n \ (n = 0, 1, \ldots, 7) \quad \text{if} \quad \frac{b}{a} = 1 - \frac{n}{10}.$$
Surface brightness profile of most elliptical galaxies can be fit well by the $R^{1/4}$ (or de Vaucouleurs) law,

$$I(R) = I_0 \exp\left(-\xi^{1/4}\right) \quad \text{with} \quad \xi \equiv R/R_*, $$

$R_*$: a characteristic radius which scales with the azimuthal angle $\phi$ (relative to the major axis) as $R_*^{-1} \propto 1 + [(1 - \varepsilon)^2 - 1] \cos^2 \phi$.

If $R_*$ is taken to be the effective radius $R_{\text{eff}}$ within which half of the total luminosity is contained: $\int_0^{R_{\text{eff}}/R_*} \zeta \exp(-\zeta^{1/4}) d\zeta = \frac{1}{2} \int_0^\infty \zeta \exp(-\zeta^{1/4}) d\zeta$, then

$$I(R) = I_{\text{eff}} \exp\left[-7.67 \left(\xi^{1/4} - 1\right)\right] \quad \text{with} \quad \xi \equiv R/R_{\text{eff}},$$

where $I_{\text{eff}}$ is the surface brightness at $R_{\text{eff}}$. The value of $R_{\text{eff}}$ is usually quoted as that of the semi-major axis.
Kinematic Properties

Big ellipticals not supported by angular momentum, but by random motion. Importance measured in terms of the ratio between the maximum line-of-sight velocity $v_m$ (relative to the mean velocity of all stars in the galaxy) and the mean velocity dispersion $\bar{\sigma}$: Flattening caused by rotation: $v_m/\bar{\sigma} \approx \sqrt{\epsilon/(1-\epsilon)}$. 
Gas Content and stellar population

Little cold gas and dust; but extended X-ray halos of hot gas.

Not much current star formation.

Giant ellipticals contain mainly old (Population II) stars. Most of the stars form early.

Some elliptical galaxies contain small gaseous disks ($M \sim 10^6$-$10^8 M_\odot$) near their centers.
Color-Magnitude Relation, Metal Content
Scaling Relations

Basic properties: $R_{\text{eff}}$, $L$ (or $I_{\text{eff}}$), and $\sigma_0$ (central velocity dispersion). Each of these parameters covers a large range, but they are tightly correlated:
The fundamental plane relation:

\[ \log R_{\text{eff}} = a \log \sigma_0 - b \log \langle I \rangle_{\text{eff}} + \text{constant}. \]

where \( a \approx 1.24, \ b \approx -0.82 \) in the optical, and \( a \approx 1.53, \ b \approx -0.79 \) in near infrared.

The Faber-Jackson relation: \( L \propto \sigma^4 \)

The \( D_n - \sigma_0 \) relation: \( D_n \propto \sigma^{1.2} \), where \( D_n \) is the physical radius at some isophotal level.

Both are projections of the fundamental plane
Physical explanation of the fundamental plane

The fundamental-plane relation is interpreted in terms of the virial theorem:

\[
\frac{GM}{\langle R \rangle} = k_E \frac{\langle v^2 \rangle}{2},
\]

\(\langle R \rangle\): average radius such that the l.h.s. is the mean specific potential energy, \(\langle v^2 \rangle\) is so defined that \(\langle v^2 \rangle/2\) is the mean specific kinetic energy, \(k_E\) is equals to 2 for a system in equilibrium.
Expressing $\langle R \rangle$ and $\langle v^2 \rangle$ in terms of observables $R_e$ (e.g. $R_{\text{eff}}$), and $\sigma_e$ (e.g. $\sigma_0$):

$$R_e = k_R \langle R \rangle; \quad \sigma_e = k_V \sqrt{\langle v^2 \rangle},$$

writing $L = M/(M/L)$, $L = k_L I_e R_e^2$ ($k_R$, $k_V$ and $k_L$ are dimensionless quantities), we obtain

$$R_e = C_R \sigma_e^2 I_e^{-1} \left( \frac{M}{L} \right)^{-1}, \quad C_R \equiv \frac{k_E}{2 G k_R k_L k_V^2}; \quad L = C_L \sigma_e^2 I_e^{-1} \left( \frac{M}{L} \right)^{-2}, \quad C_L = k_L C_R^2.$$
If elliptical galaxies are homologous (i.e. they have the same density and velocity profiles so that $C_R$ and $C_L$ are the same for all of them), and if they all have the same $(M/L)$, then the virial relation defines a plane in the $\log R_e$-$\log \sigma_e$-$\log I_e$ space, or in the $\log L$-$\log \sigma_e$-$\log I_e$ space.

In the more general case where $(M/L)$, $C_R$ and $C_L$ are power laws of $I_e$, $\sigma_e$ and $R_e$ (or $L$), the virial theorem still defines a plane in the parameter space, but the plane will be tilted with respect to the one with constant $(M/L)$ and $C_R$ (or $C_L$). The observations on the FP can therefore provide important information on the formation and evolution of elliptical galaxies.
Assume that ellipticals are homologous, the observed FP implies a change of $(M/L)$ with $L$ and $\langle I \rangle_{\text{eff}}$ as

$$(M/L) \propto L^{(2-a)/2a} \langle I \rangle_{\text{eff}}^{-1/2-(1+2b)/a}.$$ 

The observed values of $a$ and $b$ implies that

$$(M/L) \propto L^{0.31} \langle I \rangle_{\text{eff}}^{0.02} \text{ (in optical)}, \quad (M/L) \propto L^{0.15} \langle I \rangle_{\text{eff}}^{-0.12} \text{ (in near-infrared)}.$$ 

Other possibilities: elliptical galaxies may not be homologous in their structures, and so $k_R$ and $k_V$ may depend on $\sigma_e$ and $I_e$. 
The Morphology-Density Relation

Given that galaxies have different intrinsic properties, one natural question is whether these properties are correlated with the environments. The answer is yes for some properties. An important example is the morphology-density relation.
Elliptical Galaxies as Collisionless Stellar Systems

Since elliptical galaxies consist mainly of stars, one may hope to understand their structural and kinematic properties by treating them as self-gravitating stellar systems.

The time it takes for an $N$-body system to relax to an equilibrium state through two-body scattering is $t_{\text{relax}} \sim (0.1N/\ln N)t_{\text{cross}}$, where $t_{\text{cross}}$ is the typical time for a star to cross the system once. This time is very long for galaxies, and so the stellar component in a galaxy can be assumed to be collisionless.
Simple Dynamical Models

The structure and kinematics given by the phase-space distribution function $f(x,v,t)$:

$$\rho_\star(x) = m \int f(x,v) \, d^3 v, \quad \rho(x) = \frac{M}{L} \rho_\star(x).$$

$$\langle v_i \rangle (x) = \frac{m}{\rho_\star(x)} \int v_i f(x,v) \, d^3 v, \quad \langle v_i v_j \rangle (x) = \frac{m}{\rho_\star(x)} \int v_i v_j f(x,v) \, d^3 v.$$ 

Observables: surface density, mean velocity along a line-of-sight, and mean velocity dispersion, at a position $(x,y)$

$$\Sigma = \int \rho_\star(x) \, dz, \quad \langle v_z \rangle = \frac{1}{\Sigma} \int \rho_\star(x) \langle v_z \rangle (x) \, dz, \quad \langle \sigma_z^2 \rangle = \frac{1}{\Sigma} \int \rho_\star(x) \langle v_z^2 \rangle (x) \, dz - \langle v_z \rangle^2.$$
The Isothermal Sphere

The distribution function:

\[
f(E) = \frac{(\rho_0/m)}{(2\pi\sigma^2)^{3/2}} \exp\left(\frac{E}{\sigma^2}\right),
\]

where \(\sigma^2\) is a constant. The density is \(\rho(r) = \rho_0 \exp(\Psi/\sigma^2)\), and Poisson’s equation:

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Psi}{dr} \right) = -4\pi G \rho_0 \exp\left(\frac{\Psi}{\sigma^2}\right).
\]

With the boundary condition \(\rho(0) = \rho_0\), \(d\Psi/dr = 0\), the above equation can be integrated. The resulted density profile is characterized by a *King radius*:

\[
r_0 = \frac{3\sigma}{\sqrt{4\pi G \rho_0}}.
\]
\[ \rho(r) \approx \frac{\rho_0}{[1 + (r/r_0)^2]^{3/2}} \text{ (for } r \ll 2r_0) , \quad \rho(r) = \sigma^2/(2\pi Gr^2) \text{ (for } r > 10r_0) \]

Note that the total mass involved is infinite!
King Model

\[ f(\mathcal{E}) = \begin{cases} 
(\rho_0/m)(2\pi\sigma^2)^{-3/2} \left( e^{\mathcal{E}/\sigma^2} - 1 \right) & (\mathcal{E} > 0) \\
0 & (\mathcal{E} \leq 0)
\end{cases} \]

Note that no particles have \( \mathcal{E} \leq 0 \). The density is

\[ \rho(r) = 4\pi \int f(\mathcal{E})v^2 \, dv \]

\[ = \rho_0 \left[ e^{\Psi/\sigma^2} \text{erf} \left( \frac{\sqrt{\Psi}}{\sigma} \right) - \sqrt{\frac{4\Psi}{\pi\sigma^2}} \left( 1 + \frac{2\Psi}{3\sigma^2} \right) \right], \quad (1) \]

which in Poisson’s equation gives an equation for \( \Psi \). This equation for \( \Psi \) can be integrated numerically with the boundary conditions \( \Psi = \Psi_0 \) and \( (d\Psi/dr) = 0 \) at \( r = 0 \). The observed light distribution in bright ellipticals can be fit by King models with \( \Psi(0)/\sigma^2 \approx 10.9 \).
Models with $r^{-4}$ outer profiles

$$\rho(r) = \frac{(3-\gamma)M}{4\pi} \frac{a}{r^\gamma(r+a)^{4-\gamma}}.$$  

For $\gamma = 1$ (the Hernquist model), the

$$f(\tilde{E}) = M \left[ \frac{3 \sin^{-1} q + q \sqrt{1-q^2}(1-2q^2)(8q^4)}{8 \sqrt{2}\pi^3 a^3 v_g^3 (1-q^2)^{5/2}} \right],$$

$q \equiv \sqrt{\tilde{E}}$ and $v_g \equiv \sqrt{\frac{GM}{a}}$. 
The fact that the collapse of an N-body system with cold and clumpy initial configuration generally leads to the formation of an elliptical-like remnant with $R^{1/4}$-profile suggests that ellipticals may have formed from the collapses of cold and clumpy stellar systems. To understand the formation of elliptical galaxies, it is important to understand how such initial conditions are generated.
Three possibilities.

1. The formation of an elliptical by a major merger of two disk galaxies (perhaps with bulges) of comparable masses. In this case, most stars in the elliptical galaxy are formed in its progenitor disks.

2. The formation of an elliptical galaxy by a sequence of mergers of galaxies with individual masses much smaller than the remnant elliptical. As in the major-merging scenario, most stars in the remnant are formed in its progenitors.

3. The formation of an elliptical through the collapse of a large chunk of gas cloud. In this model, stars are formed as the cloud contracts and fragments.

The last possibility is usually called the monolithic-collapse scenario, while the first two are called the merging scenarios.
Monolithic collapse versus merging


Ellipticals and bulges formed in some early epoch through dissipational collapse of gas clouds and evolved only passively ever since.

Motivation: Ellipticals appear to be well developed stellar systems with old stellar populations.

Numerical simulations show that, in order to produce an ellipsoidal object, the initial cloud must be very clumpy so that the gas cools off quickly and makes stars in subclumps before the main body collapses; the gas cloud must also be relatively free of angular momentum because otherwise it will spin up as it collapses.

Problems: It is not based on any well-motivated initial conditions.
Merging Scenario: Toomre (1977):

Stars are formed in galactic disks and that all ellipticals are formed by the mergers of stellar disks.

Motivation: (1) Star-formation activities in local Universe are largely in spiral galaxies and starbursts; galaxy mergers do exist in the local universe; (2) Numerical simulations show that mergers of two stellar disks can produce galaxies with properties similar to present ellipticals.

Some problems: (1) Remnants have $\varepsilon \sim 0.3-0.7$, which appears to be too elongated to account for $E_0$, $E_1$ and $E_2$; (2) Although surface density profile of a merger product is well-fitted by $R^{1/4}$-profile, the merger remnant has a prominent core of constant surface density, while cores of observed ellipticals are much smaller.

Solution: (1) Progenitors contain bulges; (2) multiple mergers; (3) progenitors contain gas.
Similarities and differences

Not much difference so far as merging is concerned. Even in the monolithic-collapse, there must also be mergers among the cloud subclumps as they cools and form stars.

In the hierarchical model, the material in present galaxies was all in the form of uniformly distributed gas at high redshift, and so both dissipational collapses and mergers of stellar systems must be relevant to the formation of elliptical galaxies.

The real question is when and how mergers happen.
1. In the monolithic collapse model, there was, for each elliptical galaxy, a well-defined formation time after which the galaxy remained more or less constant in mass, size and shape, while in a hierarchical model a galaxy may grow continuously by accretion and merger.

2. In the monolithic collapse model, stars in an elliptical galaxy formed over a short timescale during the collapse (i.e. in a starburst), while in a hierarchical model, stars might form at different times.
A monolithic collapse is not required for explaining the old stellar age in ellipticals, because stars could have formed before they ensembled into the final galaxy.

A monolithic collapse is neither required for explaining the structure of an elliptical because, as we have seen, violent relaxation of clumpy and cold collapse can quickly leads to a configuration resembling an elliptical galaxies.

However, the monolithic collapse model represents the simplest possible model for the formation of an elliptical – only the age of the galaxy is required to predict its spectral and chemical properties – and so it can be used as a reference to compare with other more complicated models.
Observational Tests

Models for the formation of elliptical galaxies differ mainly in the following aspects:

1. Star formation history: while monolithic-collpase model predicts an early starburst for each elliptical, hierarchical models predict an extended period of star formation for each elliptical galaxy.

2. Assembling history: while monolithic-collpase model predicts that an elliptical assembled most of its stars during a short time in an early epoch, hierarchical models predict an extended period for the assembly.

3. Progenitor properties: while monolithic-collpase model predicts the progenitor of an elliptical galaxy to be a starburst from the collapse of a gaseous cloud, elliptical galaxies in hierarchical models may have a diversity of progenitors (stellar disks, bulges, gaseous disks and clouds).
Constraint from Phase-Space Density

The maximum value of the coarse-grained phase-space density \( f_c \) in a system should not increase during collisionless evolution. If giant elliptical galaxies are produced by mergers of stellar disks, then the maximum phase-space density of giant ellipticals should not exceed that of their disk progenitors.

\[
dN = f(v, x) d^3 x d^3 v.
\]
The size distribution

- Merger of present-day spirals does not work

- Repeated merger may work:

$$\frac{M^2}{R} = \frac{M_1^2}{R_1} + \frac{M_2^2}{R_2} + \frac{f_{\text{orb}}}{\alpha} \frac{M_1 M_2}{R_1 + R_2}$$

$$f_{\text{orb}}/\alpha \sim 5/3$$
Frequency of Globular Clusters

Ellipticals are found to have more globular clusters per unit mass than spirals. 

\[ S_N: \text{(number of globular clusters)/(galaxy luminosity in } M_V = -15) : \]

\[ S_N \sim 5 \text{ for cluster ellipticals, } S_N \sim 3 \text{ for field ellipticals, } S_N \sim 0.5 - 1 \text{ for spirals.} \]

Is this a serious problem for the merger scenario?

If ellipticals are the merger remnants of spirals, \( S_N \) should be the same. But this argument applies only to mergers of pure stellar disks. If gas is involved in the mergers, the formation of globular clusters may be enhanced in starbursts induced by mergers. The observed specific frequency of globular clusters in interacting systems appears to be higher than that in spirals.
Hierarchical models with high $\Omega_0$ predict that many cluster ellipticals might have been assembled quite late, while in a passive evolution model elliptical galaxies are assumed to have assembled at $z \gg 1$, so that their number density at $z \sim 1$ should be the same as at present time. Therefore, a test to distinguish theoretical models can be made by studying the number density of bright elliptical galaxies at high redshifts.

Results not yet conclusive!
HST observations
Spectral Evolution

- The passive evolution model
- Hierarchical merging model

Different star formation history may lead to difference in spectral properties. But as long as stars form early in the hierarchical model, the difference is small.
From Abundance Ratios

- $\langle \text{Fe} \rangle$ vs. $\text{CN}_1$
- $\langle \text{Fe} \rangle$ vs. $\text{Ca}4455$
- $\langle \text{Fe} \rangle$ vs. $\text{Mg}_2$
- $\langle \text{Fe} \rangle$ vs. $\text{Na} D$