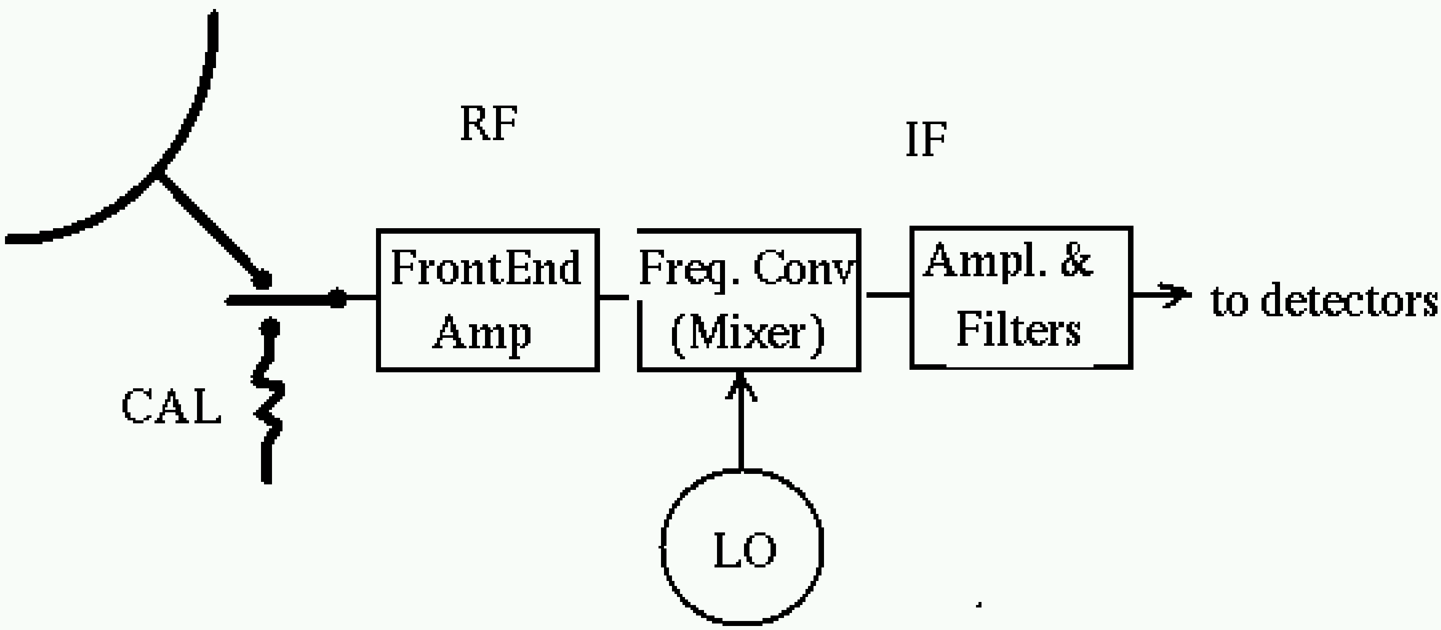
Lecture 2: Noise in Receivers

Receiver System:



- Horn and CAL Load
- FrontEnd (Low Noise) Amplifier
- Frequency Converter (Mixer)
- IF Amplifier and Filters
- Detectors

Fundamental Noise Limit

- Gaussian statistics: $\sigma(N) = \sqrt{N}$
- $N = \tau/\Delta t = \tau \times \Delta\nu$ independent samples
- Uncertainty (or "thermal noise") in a single sample T_{sys}:

$$\boxed{\frac{\Delta T}{T_{sys}} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{\Delta\nu\tau}}}$$

(see RW p62-63 for full derivation)

Example: $T_{sys} = 150$ K, $\nu = 100$ GHz, $\Delta\nu = 1$ MHz, $\tau = 1$ minute

$$\Delta T = \frac{150}{\sqrt{10^6 \times 60}} = 0.02 \text{ K}$$

Receiver Stability

- signal measured by the detector: $W = k(T_A + T_{sys})G\Delta\nu$
- gain variation leads to: $W+\Delta W = k(T_A+T_{sys})(G+\Delta G)\Delta\nu$
- variation in T_{sys} leads to: $W+\Delta W = k(T_A+T_{sys}+\Delta T)G\Delta\nu$

then, $T_{sys}\Delta G = \Delta T_{sys} G$

$$\boxed{\frac{\Delta T}{T_{sys}} = \frac{\Delta G}{G}}$$

Combined in quadrature with thermal noise,

$$\boxed{\frac{\Delta T}{T_{sys}} = \sqrt{\frac{1}{\Delta\nu\tau} + (\frac{\Delta G}{G})^2}}$$

Dicke Switching

A receiver is switched between antenna (T_A) and a calibration load R (T_R):

- signal measured on the antenna:
  $W_A = k(T_A + T_{sys})G\Delta\nu$
- signal measured on the load:
  $W_R = k(T_R + T_{sys})G\Delta\nu$
- differencing the two signals:
  $W_A - W_R = k(T_A - T_R)G\Delta\nu$

$$k(T_A - T_R)(G + \Delta G)\Delta\nu = k(T_A + \Delta T - T_R)G\Delta\nu$$

$$\frac{\Delta T}{T_{sys}} = \left(\frac{\Delta G}{G}\right)\left(\frac{T_A - T_R}{T_{sys}}\right)$$

If $(T_A - T_R) \ll T_{sys}$, then the gain variation can be removed!

Receiver Calibration

In principle, one can derive T_{sys} from W_A by setting $T_A = 0$ if G is known. In practice, T_{sys} is derived by using two different loads with known temperature:

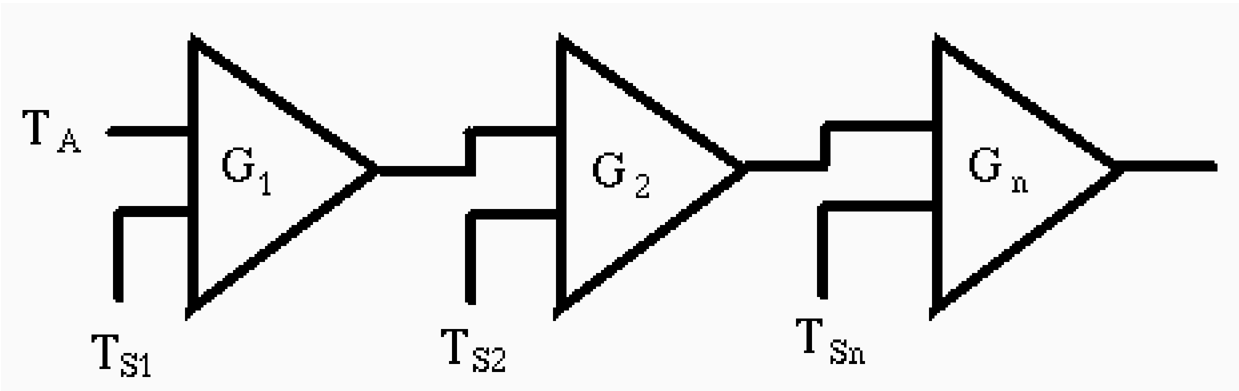
- signal measured with T_1:
  $W_1 = k(T_1 + T_{sys})G\Delta\nu$
- signal measured with T_2:
  $W_2 = k(T_2 + T_{sys})G\Delta\nu$
- "Y factor": $Y \equiv \frac{W_1}{W_2} = \frac{T_1+T_{sys}}{T_2+T_{sys}}$

$$\boxed{T_{sys} = \frac{T_1 - T_2 Y}{Y - 1}}$$

Example: $T_1 = 300$ K (ambient load), $T_2 = 3$ K (sky)

Cascading of Amplifiers

To amplify power by 80-140 db (10^{8-14})



$$G = \Pi G_i$$

- $P_0 = kT_A$
- $P_1 = (P_0 + kT_{S1})G_1 = k(T_A + T_{S1})G_1$
- $P_2 = (P_1 + kT_{S2})G_2 = k([T_A + T_{S1}]G_1 + T_{S2})G_2$
- $P_n = (P_{n-1} + kT_{Sn})G_n$

Cascading of Amplifiers (cont.)

To rewrite as $P_n = k(T_A + T_S)G$,

$$T_S = T_{S1} \ + \ \frac{1}{G_1}T_{S2} \ + \ \frac{1}{G_1G_2}T_{S3} \ + \ ... \\ + \frac{1}{G_1G_2....G_{n-1}}T_{Sn}$$

> To minimize T_S, keep T_{S1} as small as possible for all $G_i > 1$.

Low Noise Front-End Amplifiers:

- **maser amplifiers** ($\frac{n_2}{n_1} = \frac{g_1}{g_2}e^{-\Delta E/kt}$)
- **parametric amplifiers (oscillating capacitance)**
- **transister amplifiers (FET, HEMT)**