Astronomy 452 Astrophysics II: Galaxies

Homework I (Due 2/18/2009, in class)

(i) Two vectors $\mathbf{A}(t)$ and $\mathbf{B}(t)$, both are functions of t. Prove that

$$\frac{d(\mathbf{A}\cdot\mathbf{B})}{dt} = \mathbf{A}\cdot\frac{d\mathbf{B}}{dt} + \frac{d\mathbf{A}}{dt}\cdot\mathbf{B}\,;$$

and

$$\frac{d(\mathbf{A}\times\mathbf{B})}{dt} = \mathbf{A}\times\frac{d\mathbf{B}}{dt} + \frac{d\mathbf{A}}{dt}\times\mathbf{B}\,.$$

(ii) The angular momentum of a point mass of mass m moving with a velocity $\mathbf{v}$ at a position $\mathbf{r}$ is defined as

$$\mathbf{L} = m\mathbf{r}\times\mathbf{v}$$

Prove that the rate of change of $\mathbf{L}$, $d\mathbf{L}/dt$, is equal to the torque: $\mathbf{T} = \mathbf{r}\times\mathbf{F}$, where $\mathbf{F}$ is the force on the mass.

(iii) The surface mass density (i.e. mass per unit area) of a galaxy disk roughly follows the exponential profile:

$$\Sigma(R) = \Sigma_0 e^{-R/R_d}\,,$$

where Σ_0 (the central surface density) and R_d (the scalelength) are constant. What is the total mass M of the disk (express it in terms of Σ_0 and R_d).

(iv) As we will learn later, galaxies possess extended dark matter halos with density profiles that roughly follow the following form:

$$\rho(r) = \rho_0\left(\frac{r_0}{r}\right)^2\,,$$

where r_0 and ρ_0 are constant, and r is the distance from the halo center. What is the mass of dark matter within a sphere of radius R around the center?

(v) The most powerful telescopes available at the present time can observe an object down to about 30th magnitude in the B-band. To what distance can such a telescope observe a star like the sun? What kind of energy flux is observed at the telescope in terms of the solar constant? To which distance can this telescope observe a galaxy like the Milky Way whose luminosity is about 10^{10} times that of the sun?

(vi) What is the magnitude of the merger of two galaxies that have magnitudes m_1 and m_2, respectively?

(vii) Two stars, 1 and 2, have the same V-magnitude: $V = 7.5$, but have different B-magnitudes $B_1 = 7.2$, $B_2 = 8.5$. What are the $(B - V)$ colors of these two stars? Which star is bluer? What is the flux ratio, f_1/f_2, in the B-band? If star 2 has a distance that is 10 times that of star 1, what are the luminosity ratios: L_1/L_2, in both B- and V-bands?

(viii) A gas cloud has a temperature of 5000 K. Estimate the width of hydrogen H-alpha line with intrinsic wavelength $\lambda = 656$ nm. Hint: the typical velocity of hydrogen atoms in a gas of temperature T is about $(kT/m_{\rm H})^{1/2}$, where k is Boltzmann constant, $m_{\rm H}$ is the mass of a hydrogen atom (approximately that of a proton).