

Astronomy 452 Astrophysics II: Galaxies

Homework IV (Due Monday, 5/5/2008, in class)

- (i) Show that the $R^{1/4}$ profile (de Vaucouleurs profile),

$$I(R) = I_{\text{eff}} \exp \left[-7.67 \left((R/R_{\text{eff}})^{1/4} - 1 \right) \right],$$

yields a total luminosity

$$L = 8! \frac{e^{7.67}}{(7.67)^8} \pi R_{\text{eff}}^2 \approx 7.22 \pi R_{\text{eff}}^2 I_{\text{eff}}.$$

[Hint: $\int_0^\infty e^{-t} t^7 dt = \Gamma(8) = 7!$]

- (ii) When an E0 galaxy with stellar density $n(r)$ is viewed from a great distance along the axis z , show that the surface density at distance R from the center is

$$\Sigma(R) = 2 \int_0^\infty n(r) dz = 2 \int_R^\infty \frac{n(r) r dr}{\sqrt{r^2 - R^2}}.$$

If $n(r) = n_0 (r_0/r)^\alpha$, show that as long as $\alpha > 1$ we have

$$\Sigma(R) = 2n_0 r_0 (r_0/R)^{\alpha-1} \int_1^\infty \frac{x^{1-\alpha}}{\sqrt{x^2 - 1}} = \Sigma(R = r_0) (r_0/R)^{\alpha-1}.$$

What is the meaning of $\Sigma(R = r_0)$? What happens if $\alpha < 1$?

- (iii) Suppose the potential of an elliptical galaxy is given by

$$\Phi(x, y, z) = \frac{1}{2} \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right).$$

- (1) What is the density distribution within the galaxy?
 - (2) What is the gravitational acceleration of a particle at (x, y, z) ?
 - (3) Solve the equation of motion, and discuss how stars move in such a potential well.
 - (4) Suppose $\omega_x < \omega_y < \omega_z$. What do you think the shape of the galaxy is?
- (iv) Assume that the mass-to-light ratio, M/L and the velocity dispersion are roughly constant throughout an elliptical galaxy, that there is no dark matter, and that the galaxy can be considered as a uniform sphere.
- (1) Show that the total kinetic energy is $K = 3M\sigma^2/2$, where M

is the mass of the galaxy, σ is the 1-dimensional velocity dispersion, and the total potential energy is $W = -3GM^2/(5R)$, where R is the radius of the sphere.

(2) Use the virial theorem to show that $M = 5\sigma^2 R/G$.

(3) If all elliptical galaxies have similar structure described by the $R^{1/4}$ profile, show that we would have $M \propto \sigma^2 R_{\text{eff}}$ and that $L \propto I_{\text{eff}} R_{\text{eff}}^2$, so that the mass-to-light ratio is $M/L \propto \sigma^2/(I_{\text{eff}} R_{\text{eff}})$.

(4) Show that if all ellipticals have the same mass-to-light ratio, M/L , and the same effective surface brightness, then they follow the Faber-Jackson relation, $L \propto \sigma^4$.

(5) Suppose the observed Fundamental Plane Relation is $R_{\text{eff}} \propto \sigma^{1.2} I_{\text{eff}}^{-0.8}$. Show that this would imply a mass to light ratio, $M/L \propto \sigma^{0.5} R_{\text{eff}}^{0.25} \propto M^{0.25}$.