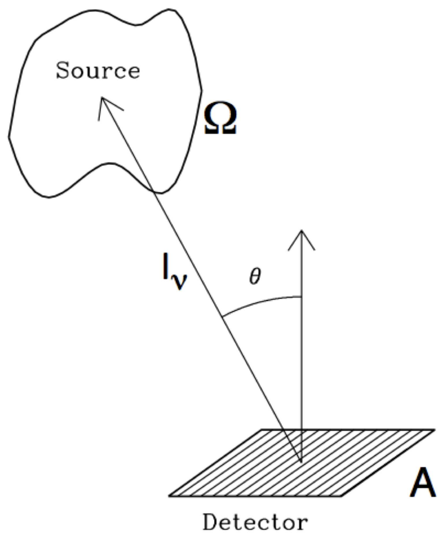
Lecture 1: Basic Concepts

Cosmic Signal

- voltage induced on a detector or power received by a telescope.
- no real information – Gaussian random noise in most cases.
- in a bandwidth of $\Delta\nu$, a random pulse would last $\sim 1/\Delta\nu$.
  - $\rightarrow$ measurements are made as ensemble averages and time averages

Spectral Power Flux Density, S_ν



Energy dE flows through area dA in time dt in the frequency range ν and $\nu + \Delta\nu$, originating from a source with solid angle dΩ, in the direction θ away from the direction normal to the detector plane A. Then

$$dE_\nu = I_\nu cos\,\theta\, dA\, d\Omega\, dt\, d\nu$$

Power : $dP_\nu = \frac{dE_\nu}{dt} = I_\nu cos\,\theta\, dA\, d\Omega\, d\nu$

and specific intensity ("spectral brightness") is

$$I_\nu = \frac{dP_\nu}{cos\,\theta\, dA\, d\Omega\, d\nu}$$

(note: I_ν is independent of distance.)

For a source with finite Ω, total energy arriving at the detector is

$$S_\nu = \int_\Omega I_\nu(\theta,\phi)\, cos\,\theta\, d\Omega \quad (1)$$

$$= \int_\Omega I_\nu(\theta,\phi)\, d\Omega \quad \text{(for small } \theta\text{)} \quad (2)$$

This "flux density" is in units of

$$1 Jy = 10^{-26}\ W\ m^{-2}\ Hz^{-1} \quad (3)$$

$$= 10^{-23}\ erg\ s^{-1}\ cm^{-2}\ Hz^{-1} \quad (4)$$

Note that $S_\nu \propto (1/D)^2$ and

Spectral luminosity : $L_\nu \equiv 4\pi D^2 S_\nu$

Bolometric luminosity : $L_{bol} \equiv \int_\nu L_\nu d\nu$

<u>Power Received</u>: $P_r = \frac{1}{2} A S_\nu \Delta\nu$

The factor of 1/2 arises because each detector is sensitive to only a single polarization.

Planck Function

$$B_\nu = \frac{2h\nu^3/c^2}{exp(h\nu/kT) - 1} = I_\nu$$

R-J: $\frac{h\nu}{kT} \ll 1$

$$B_\nu = \frac{2\nu^2}{c^2} kT$$

Brightness Temperature

$$T_B = \frac{c^2}{2k\nu^2} I_\nu$$

Nyquist Theorum and Noise

Average power per unit bandwidth received by a resister R:

$$P = \; < IV > = \frac{V^2}{2R} \tag{5}$$

$$= \frac{1}{4R} < V_N^2 > \tag{6}$$

From the random walk analysis,

$$< V_N^2 >= 4RkT$$

$$\rightarrow P_N = kT\Delta\nu$$

$$\text{Noise Temperature} : T_N = \frac{P_N}{k\Delta\nu}$$

System Temperature:

$$T_S = \frac{P_S}{k\Delta\nu}$$

($T_{sys} \sim 100 - 500$ K for FCRAO 14-m)

Antenna Temperature:

$$T_A = \frac{P_A}{k\Delta\nu}$$

($T_A \sim 10$ K for CO from GMCs)

Signal-to-Noise Ratio:

$$S/N = C\frac{T_A}{T_S}\sqrt{\Delta\nu\tau}$$