II. Practical Issues

RF Interferometer:

cross-correlation output from the correlator:

$$r(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} v_1(t) v_2(t-\tau) dt$$

In general, for $\tau_g = \vec{B} \cdot \vec{S_0}/c$,

$$r(\tau_g) = A_0 \Delta\nu |V| cos(2\pi \vec{B_\lambda} \cdot \vec{S_0} - \phi_s)$$

- amplitude $|V|$ relates to source strength
- phase ϕ_s relates to source location wrto $\vec{S_0}$
- antenna area A_0 and bandwidth $\Delta\nu$ are instrumental

Wiener-Khinchin relation:

$$\int_{-\infty}^{+\infty} r(\tau) e^{-i2\pi\nu\tau} d\tau = |H(\nu)|^2$$

$$and \int_{-\infty}^{+\infty} |H(\nu)|^2 e^{i2\pi\nu\tau} d\tau = r(\tau)$$

where $H(\nu)$ is amplitude (voltage) response and $|H(\nu)|^2$ is power spectrum (**instrumental bandpass**).

Finite Bandwidth:

Real interferometers have finite bandwidths, and the correlator output is an integral over frequency.

$$V = \int I_\nu [\frac{1}{\Delta\nu} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} e^{-i2\pi\nu\tau_g} d\nu] d\Omega$$

$$= \int I_\nu e^{-i2\pi\nu_0\tau_g} \frac{sin(\pi\tau_g\Delta\nu)}{\pi\tau_g\Delta\nu} d\Omega$$

To avoid loss of amplitude (sensitivity), we must track delay.

Delay Tracking ("fringe stopping"):

- reduce "bandwidth smearing"
- 2-D FT requires $\tau = 0$
  - remove instrumental delay offset τ_i
  - remove geometrical delay τ_g

FOV and Resolution:

- Field of View = λ/D

Example: $\nu = 230$ GHz, $D = 6$ m
$FOV = \frac{3\times10^{10}}{(230\times10^9)(600)} = 2.2\times10^{-4} = 45''$

- Resolution: $\theta_{syn} \sim \lambda/B_{max}$

Example: $\nu = 230$ GHz, $B_{max} = 300$ m
$\theta_{syn} = \frac{3\times10^{10}}{(230\times10^9)(30000)} = 4.3\times10^{-6} = 0.9''$

Sensitivity:

$$\sigma = \frac{2kT_{sys}}{\eta_q A\sqrt{N_A(N_A-1)\tau\Delta\nu}}$$

Example: $T_{sys} = 200$ K, $N_A = 8$, $A = \frac{\pi}{4}(600)^2$, $\Delta\nu = 4$ GHz, $\eta_q = 0.88$, $\tau = 60$ seconds

$\Rightarrow \sigma = 6$ mJy in 1 minute

Baseline and uv-Coverage:

Standard sampling requirement applies: to "map" a source, sample the uv-space as fully as possible.

uv-sampling function $\rightleftharpoons$ PSF

(i.e., poor uv-sampling limits deconvolution.)

- astronomical coordinate system fixed on source
- a fixed baseline rotates with Earth, and so does the projected baseline
  $\Rightarrow$ a fixed baseline tracks an arc in uv-space
- since $I(l, m)$ is real, $V(-u, -v) = V^*(u, v)$
  $\Rightarrow$ actually tracks two arcs in uv-space