Astro731 - Radio Astronomy

13 Lecture 13: Astrometry and Geodesy

For a connected element array

$$
\begin{aligned}
\vec{B}_\lambda &\equiv \text{baseline vector} \\
\Delta\vec{B}_\lambda &\equiv \text{“error''' in } \vec{B}_\lambda \\
\vec{S} &\equiv \text{source position} \\
\Delta\vec{S} &\equiv \text{“error''' in position}
\end{aligned}
$$

Then, the resulting phase difference is

$$
\begin{aligned}
\Delta\phi_{ij} &\equiv \Delta\phi_{ij,obs} - \Delta\phi_{ij,true} \\
&= 2\pi[(\vec{B}_\lambda - \Delta\vec{B}_\lambda)\cdot(\vec{S} - \Delta\vec{S}) - \vec{B}_\lambda\cdot\vec{S}] + \phi_{inst} \\
&= \phi_{inst} - 2\pi(\Delta\vec{B}_\lambda\cdot\vec{S} + \vec{B}_\lambda\cdot\Delta\vec{S})
\end{aligned}
$$

Therefore, “error” in baseline and “error” in source positions are equivalent.

In terms of the coordinate systems:

$$
\vec{B}_\lambda = \begin{bmatrix} X_\lambda \\ Y_\lambda \\ Z_\lambda \end{bmatrix}, \qquad \Delta\vec{B}_\lambda = \begin{bmatrix} \Delta X_\lambda \\ \Delta Y_\lambda \\ \Delta Z_\lambda \end{bmatrix}
$$

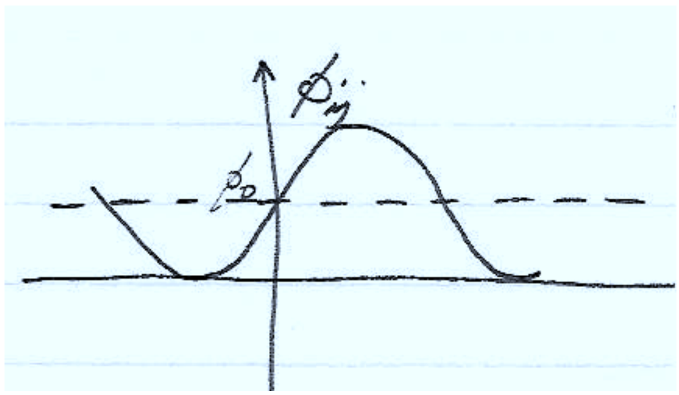
$$
\vec{S} = \begin{bmatrix} S_X \\ S_Y \\ S_Z \end{bmatrix} = \begin{bmatrix} cos\,\delta cos\,H \\ -cos\,\delta sin\,H \\ sin\,\delta \end{bmatrix}
$$

$$
\Delta\vec{S} = \begin{bmatrix} -sin\,\delta cos\,H\Delta\delta + cos\,\delta sin\,H\Delta\alpha \\ sin\,\delta sin\,H\Delta\delta + cos\,\delta cos\,H\Delta\alpha \\ cos\,\delta\Delta\delta \end{bmatrix}
$$

Then, $\phi_{ij}(H) = \phi_0 + \phi_1 cos\,\delta cos\,H + \phi_2 cos\,\delta sin\,H$

$$\text{where} \begin{pmatrix} \phi_0 = \phi_{inst} - 2\pi(\Delta Z_\lambda sin\,\delta + Z_\lambda \Delta\delta cos\,\delta) \\ \phi_1 = 2\pi(-\Delta X_\lambda + X_\lambda \Delta\delta tan\,\delta - Y_\lambda \Delta\alpha) \\ \phi_2 = 2\pi(\Delta Y_\lambda - X_\lambda \Delta\alpha - Y_\lambda \Delta\delta tan\,\delta) \end{pmatrix}$$

$\phi_{ij}(H)$ is sinusoidal in H with an offset ϕ_0.



If one source is tracked in time (H), 3 parameters (amp, phase, & offset) are derived. **If** m **sources are observed,** $3m$ **quantities are derived!**

- Each source has 2 unknowns (positions) $\Longrightarrow$ 2m unknowns
- 3 unknown baseline parameters
- 1 constant phase ϕ_{inst}

$\Longrightarrow 2m + 4$ unknowns (actually only $2m + 3$ since ϕ is a function of H)

$\Longrightarrow$ If $m \geq 3$, this is an over-determined problem and becomes possible to solve for *all* unknown quantities!

So, observe many sources and solve for all parameters simultaneously using a least mean square analysis.

13.1 Baseline Determination .

Method 1: Observe two or more astronomical standards (QSOs; $\Delta\vec{S} = 0$) over a broad range of H:

$$\begin{aligned} \phi(H) &= \phi_{inst} - 2\pi\Delta\vec{B}_\lambda \cdot \vec{S} \\ &= \phi_{inst} + 2\pi(\Delta X_\lambda cos\,\delta cos\,H - \Delta Y_\lambda cos\,\delta sin\,H - \Delta Z_\lambda sin\,\delta) \end{aligned}$$

$\Longrightarrow$ Derive ϕ_{inst}, ΔX_λ, ΔY_λ. and ΔZ_λ.

Method 2: Observe a large number of astronomical standards (QSOs; $\Delta\vec{S} = 0$) spread over a large range of $(\alpha,\ \delta)$ and examine the phase difference between adjacent pairs:

$$\begin{aligned} \Delta\phi &\equiv \phi(S_1) - \phi(S_2) \\ &= 2\pi[\Delta X_\lambda(cos\,\delta_1 cos\,H_1 - cos\,\delta_2 cos\,H_2) \\ &\quad -\Delta Y_\lambda(cos\,\delta_1 sin\,H_1 - cos\,\delta_2 sin\,H_2) \\ &\quad -\Delta Z_\lambda(sin\,\delta_1 - sin\,\delta_2)] \end{aligned}$$

$\Longrightarrow$ Get $\Delta\vec{B}_\lambda$ by χ^2 minimization over many pairs of sources.

13.2 Astrometry .

Measure phase relative to nearby astrometric calibrators ("nearby" $\equiv |\Delta\vec{S}|/|\vec{S}| \ll 1$):

$$\phi(H) = \phi_{inst} - 2\pi\vec{B}_\lambda \cdot \Delta\vec{S}$$

For an astrometric calibrator, $\phi(H) = \phi_{inst}$ as $\Delta\vec{S} = 0$. So ϕ_{inst} can be determined precisely. For the nearby target source,

$$\begin{aligned} \phi(H) &= -2\pi\vec{B}_\lambda \cdot \Delta\vec{S} \\ &= 2\pi[X_\lambda(-sin\,\delta cos\,H\Delta\delta + cos\,\delta sin\,H\Delta\alpha) \\ &\quad +Y_\lambda(sin\,\delta sin\,H\Delta\delta + cos\,\delta cos\,H\Delta\alpha) \\ &\quad +Z_\lambda cos\,\delta\Delta\delta] \end{aligned}$$

Measure many astrometric calibrators and solve for $\Delta\alpha$ and $\Delta\delta$.

13.3 Geodetic Measurements .

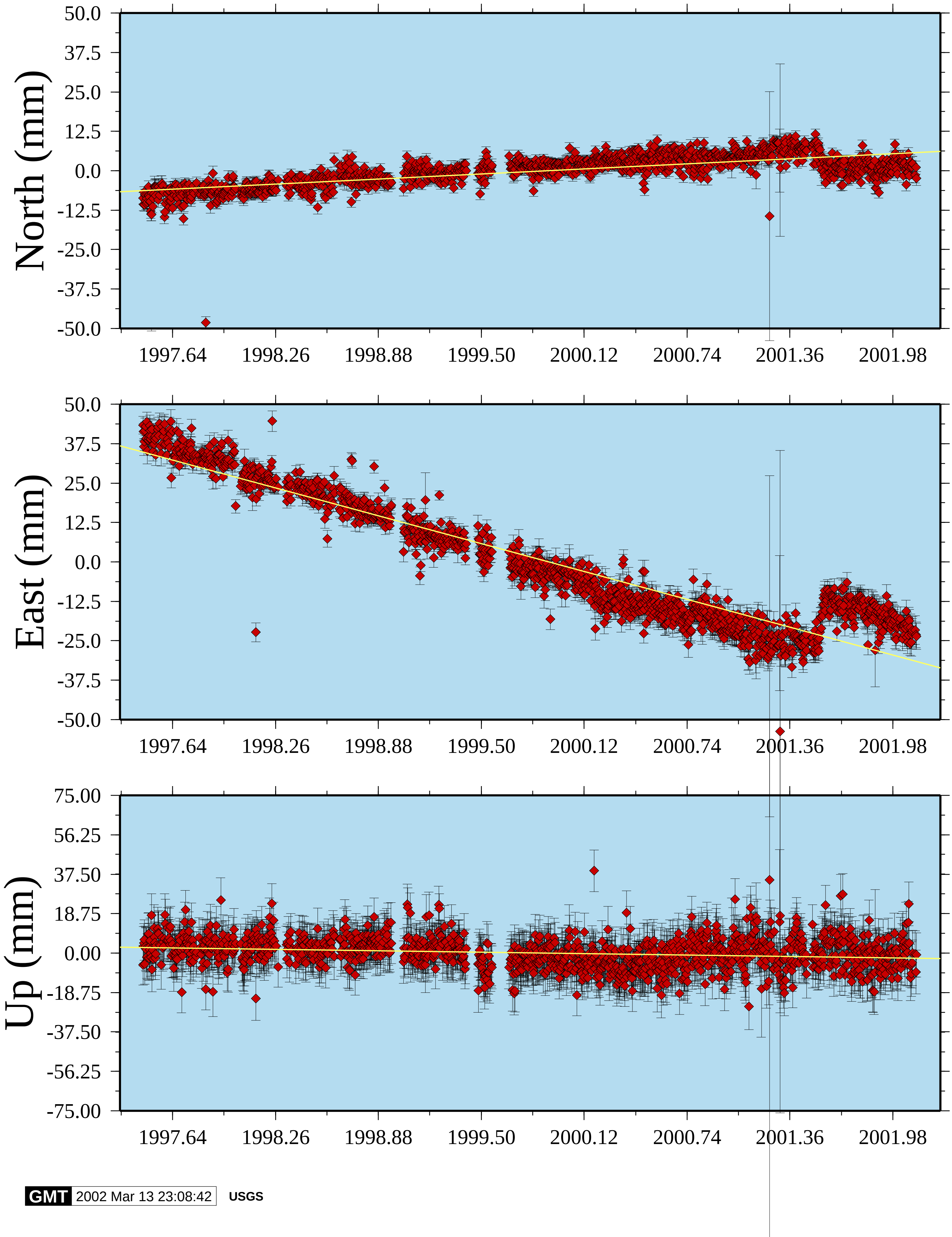
Measurements of Earth tides, tectonic plate movements, etc.

- Typically a few cm per year via VLBI
- More recently, use a cluster of GPS and array of receivers are used to improve time and spatial resolution

  $\Longrightarrow$ San Andreas Fault is moving at $\sim$ 1 ft/yr!

  (See http://pasadena.wr.usgs.gov/scign/Analysis/javaplot/TimeSeries.html)

COSO Position, Demeaned



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