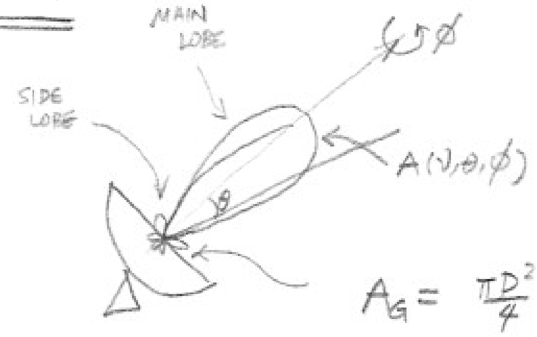
ANTENNA PATTERN (DEFINITIONS)

$I(\nu, \theta, \phi)$



"circular paraboloid"

$A_G = \frac{\pi D^2}{4}$ GEOMETRIC AREA

$$P = I(\nu,\theta,\phi)\underbrace{A(\nu,\theta,\phi)}_{\text{EFFECTIVE COLLECTING AREA}}\Delta\nu\,\Delta\Omega$$

$P_n^{\nu}(\theta,\phi) \equiv$ NORMALIZED ANTENNA RECEPTION PATTERN

$= A(\nu,\theta,\phi)/A_0$

A_0 → RESPONSE AT THE CENTER OF THE MAIN LOBE

BEAM SOLID ANGLE $\Omega_A = \iint P_n(\theta,\phi)\, d\Omega$

ANTENNA PATTERN IS A RESULT OF INTERACTIONS OF INCIDENT WAVES AT WAVELENGTHS λ,

SMALLEST COHERENT INTERACTION SCALES

$\Longleftrightarrow$ LARGES GEOMETRICAL SCALES, D.

$\theta = \frac{\lambda}{D}$

$\Omega \propto \left(\frac{\lambda}{D}\right)^2$

"THROUGHPUT" or "EFFECTIVE AREA" $\Rightarrow$ $\boxed{A_0\Omega_A = \lambda^2}$

Ex: if $P_n = 1$ everywhere (isotropic case)

$\Rightarrow \Omega_A = \iint 1\, d\Omega = 4\pi$ (maximum)

$\Rightarrow A_0 = \frac{\lambda^2}{\Omega_A} = \frac{\lambda^2}{4\pi}$ (minimum)

if $P_n = \delta(\alpha,\delta)$, $\Omega_A = 1$, $A_0 = \lambda^2$.

APERTURE EFFICIENCY

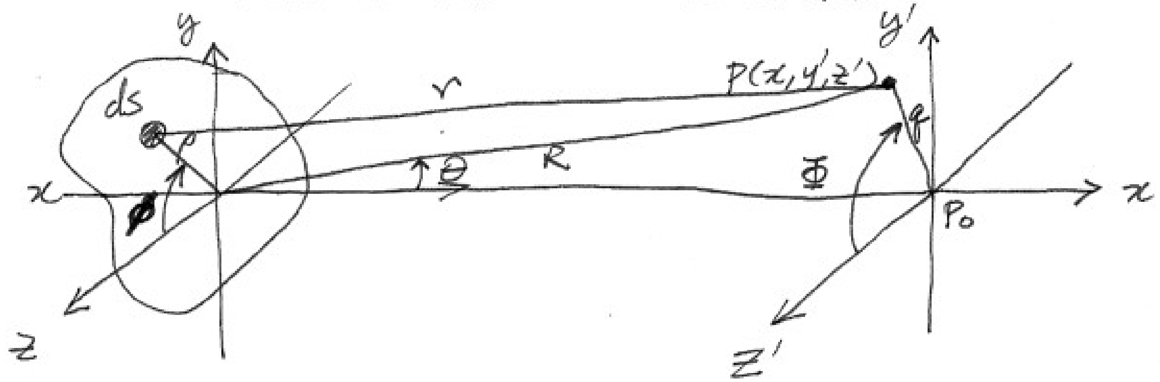
FOR AN ANTENNA WITH A WELL DEFINED PHYSICAL COLLECTING AREA A_G,

$$A_c = \eta A_G$$

where η is "APERTURE EFFICIENCY"

GEOMETRIC OPTICS

[NOTE : TRANSMISSION $\longleftrightarrow$ RECEPTION]



E - FIELD INDUCED BY ds AT $P(x, y', z')$

$$dE = \frac{\mathcal{E}_A}{|r|} e^{i(\omega t - kr)} ds$$

NET FIELD INDUCED AT $P(x, y', z')$

$$E = \iint \frac{\mathcal{E}_A}{r} e^{i(\omega t - kr)} ds$$

[NOTE: e^{-ikr} is F.T.]

FAR-FIELD OR FRAUNHOFER LIMIT : $R \gg \rho$

$$R = [x^2 + y'^2 + z'^2]^{1/2}$$

$$r = [\cancel{x}^2 + (y'-y)^2 + (z'-z)^2]^{1/2}$$

$$\Rightarrow r = R[1 + \cancel{(y^2+z^2)/R}^{\,0} - 2(yy'+zz')/R^2]^{1/2}$$

$$\simeq R[1 - 2(yy'+zz')/R^2]^{1/2}$$

$$\simeq R[1 - (yy'+zz')/R^2]$$

$$\boxed{\therefore E = \frac{\mathcal{E}_A e^{i(\omega t - kR)}}{R} \iint e^{ik(yy'+zz')/R}\, ds.}$$

EXAMPLE : CIRCULAR APERTURE w/ RADIUS a.

$$\left\{ \begin{array}{ll} z = \rho\cos\phi & y = \rho\sin\phi \\ z' = q\cos\Phi & y' = q\sin\Phi. \end{array} \right\}$$

$$ds = \rho\, d\rho\, d\phi$$

$$E = \frac{\mathcal{E}_A e^{i(\omega t - kR)}}{R} \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} e^{i(k\rho q/R)\cos(\phi-\Phi)} \rho\, d\rho\, d\phi$$

(BESSEL FUNCTION ORDER m : $J_m(u) = \frac{i^{-m}}{2\pi}\int_0^{2\pi} e^{i(mv + u\cos v)} dv$

$m=0 \Rightarrow J_0(u) = \frac{1}{2\pi}\int_0^{2\pi} e^{iu\cos v} dv$)

$$E = \frac{\mathcal{E}_A e^{i(\omega t - kR)}}{R} 2\pi \int_0^a J_0(k\rho q/R)\rho\, d\rho$$

$$\left[\frac{d}{du}[u^m J_m(u)] = u^m J_{m-1}(u) \right] \quad \left(u = \frac{kq}{R}\rho\right)$$

$$= \frac{\mathcal{E}_A e^{i(\omega t - kR)}}{R} 2\pi a^2 \left(\frac{R}{kaq}\right) J_1(kaq/R)$$

$$P = |EE^*|$$

$$= \frac{2\mathcal{E}_A{}^2 A^2}{R^2}\left[\frac{J_1(kaq/R)}{kaq/R}\right]^2$$

where A = area = πa^2

AT $q=0$, $\frac{J_1(u)}{u} = \frac{1}{2}$

$$\Rightarrow P_0 = \frac{\mathcal{E}_A{}^2 A^2}{2R^2}$$

$$\boxed{\therefore P = P_0\left[\frac{2J_1(kaq/R)}{kaq/R}\right]^2}$$