Lecture 9: Holography

Motivation

At high frequencies, antenna surface error

$$\eta_{sf} \ = \ exp[-(\frac{4\pi\epsilon}{\lambda})^2]$$

dominates the overall aperture efficiency η_A. How would one measure the surface error ϵ and improve the aperture efficiency?

Classical Methods:

- radial template
- theodolite and prisms
- modulated light beams
- microwave transmitter

Disadvantages: time consuming and difficult, limited elevation ranges, size limits, etc.

Recall: Fourier Transform pair,

$$E(x, y) \rightleftharpoons G(\alpha, \varepsilon)$$

For example,

(circular aperture) $\rightleftharpoons$ (Bessel function)

Complex illumination function and the diffraction pattern of an aperture form a Fourier Transform pair.

General Concepts:

- measurements of complex pairs of the antenna beam pattern (i.e. amplitude and phase – thus needs a phase reference)
- spatial resolution D/α is determined by angular resolution $\alpha\lambda/D$
- Nyquist sampling at λ/D (complex numbers)
- Two distinct modes:
  - single dish observations with a fixed second antenna/feed
  - interferometric observations

Observations: To map antenna diameter D at D/α resolution,

- map the beam pattern over $\alpha \times \lambda/D$
- raster or on-the-fly mapping and re-grid the data

Example: SMA at 230 GHz

$$\lambda = 1.3 \text{ mm}, D = 6 \text{ meter}$$
$$\text{desired resolution} = 5 \text{ cm}$$

$$\lambda/D = 0.0002167\, rad \sim 45''$$
$$\text{map size} = 45'' \times \frac{600}{5} = 1.5^\circ$$
$$(129 \times 129 \text{ cells})$$

Data Analysis:

- Interpolate the data in 2-D grid
- Amplitude and phase correction
- Some tapering to reduce the sidelobes
- Fourier Transform
- Some corrections and masking for diffraction fringes due to antenna edge, feedlegs, the central blockage
- Feed displacement corrections (pointing, focus, and aberrations)
- The corrected phase map is estimate of the surface error *normal to the wavefront*. Now derive the errors *normal to the antenna surface*!
- Adjust panels and start over!

Choice of Observing Frequency:

- The lower the frequency, the smaller the phase change for given ϵ, and so the higher the S/N requirement.
- If frequency too low, then diffraction effects significant
- The lower the frequency, the larger the mapping area in the sky
- T_{sys} generally higher at higher freq.
- Astronomical sources fainter at higher freq.
- No satellite beacons above 30 GHz
- Transmitters limited to low and fixed elevation

Sensitivity Requirements

- measurements over a range of angles $\alpha\lambda/D$
- a single patch contribute only $1/\alpha^2$ to total measurement
- an error ϵ contributes only $4\pi\epsilon/\alpha^2\lambda$
- signal from α^2 patches combined in the F.T.

If $S/N = R$ in the main beam, then

$$\boxed{\epsilon/\Delta\epsilon \sim R \; or \; \Delta\epsilon \sim \alpha\lambda/4\pi R}$$

Example: to measure ϵ for SMA antenna to 1 μm,

$$R = \alpha\lambda/4\pi\Delta\epsilon = \frac{120 \times 1300}{4\pi \times 1} \sim 10^4!$$