

# THERMAL RADIO EMISSION

## Plank Function

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda kT) - 1}$$

## Rayleigh-Jeans:

$$I_\nu(T) = \frac{2\nu^2}{c^2}kT = j_\nu/\kappa_\nu$$

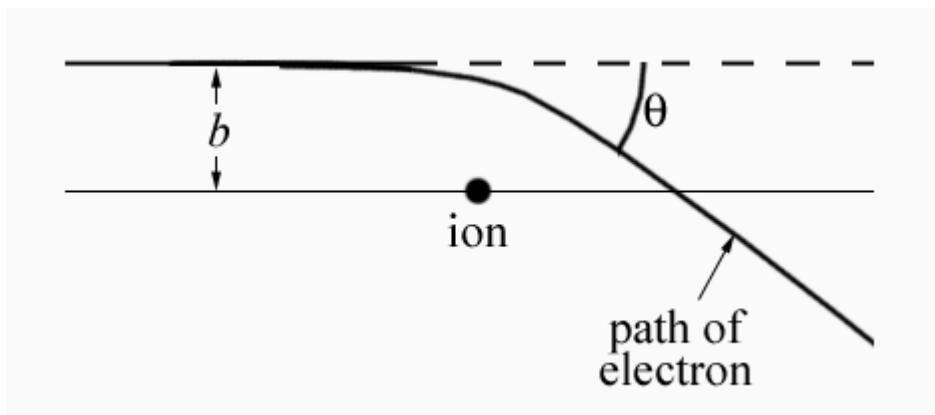
## Optical Depth: $\tau = \int \kappa_\nu ds$

$$\Rightarrow I_\nu(T) = \frac{2\nu^2}{c^2}kT(1 - e^{-\tau})$$

$$\Rightarrow S_\nu(T) = \int \frac{2\nu^2}{c^2}kT(1 - e^{-\tau})d\Omega$$

## Bremsstrahlung (free-free)

“Radiation due to the acceleration of a charged particle in the Coulomb field of another charge”



### Classical Treatment:

$$\frac{dW}{d\omega dV dt} = \frac{16e^2}{3c^3 m^2 v} n_e n_i Z^2 \ln\left(\frac{b_{max}}{b_{min}}\right)$$

### Quantum Treatment: $g_{ff} = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{max}}{b_{min}}\right)$

$$\frac{dW}{d\omega dV dt} = \frac{16\pi e^2}{3\sqrt{3}c^3 m^2 v} n_e n_i Z^2 g_{ff}$$

## Maxwellian Distribution:

Probability Distribution:

$$dP \propto \exp\left(-\frac{mv^2}{2kT}\right)d^3v \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right)dv$$

$$h\nu \leq \frac{1}{2}mv^2$$

$$\frac{dW}{d\nu dV dt} = \frac{2^5 \pi e^6}{3mc^3} \left(\frac{2\pi}{3km}\right)^{1/2} T^{-1/2} n_e n_i Z^2 e^{-h\nu/kT} g_{ff}^-$$

$$[\text{note } dw = 2\pi d\nu]$$

In cgs units,

$$\epsilon_\nu \equiv \frac{dW}{d\nu dV dt} = 6.8 \times 10^{-38} T^{-1/2} n_e n_i Z^2 e^{-h\nu/kT} g_{ff}^-$$

## Free-free Absorption coefficient:

$$\kappa_\nu = \epsilon_\nu / I_\nu = \frac{4e^6}{3kmc} \left( \frac{2\pi}{3km} \right)^{1/2} T^{-3/2} n_e n_i Z^2 \nu^{-2} g_{ff}^-$$

In cgs units,

$$\kappa_\nu = 0.018 T^{-3/2} n_e n_i Z^2 \nu^{-2} g_{ff}^-$$

## Emission Measure (EM):

$$\frac{EM}{\text{pc cm}^{-6}} \equiv \int \left( \frac{n_e}{\text{cm}^{-3}} \right)^2 d\left( \frac{s}{\text{pc}} \right)$$

## Free-free Opacity:

In fully ionized medium,  $n_e \sim n_i$  and  $Z \sim 1$ ,

$$\tau_\nu = \int \kappa_\nu ds = 0.030 \left( \frac{T_e}{\text{K}} \right)^{-3/2} \left( \frac{\nu}{\text{GHz}} \right)^{-2} \left( \frac{EM}{\text{pc cm}^{-6}} \right) g_{ff}^-$$

Altenhoff et al. (1960):  $g_{ff}^- \propto T^{0.15} \nu^{-0.1}$

$$\Rightarrow \tau_\nu = 0.082 \left( \frac{T_e}{\text{K}} \right)^{-1.35} \left( \frac{\nu}{\text{GHz}} \right)^{-2.1} \left( \frac{EM}{\text{pc cm}^{-6}} \right)$$

## What can we learn about HII regions?

1. **Temperature:** at  $\nu$  where  $\tau \gg 1$

$$S_\nu(T) = \frac{2\nu^2}{c^2} kT \Omega_s$$

2. **EM:** at  $\nu$  where  $\tau \ll 1$

$$S_\nu(T) = \tau \frac{2\nu^2}{c^2} kT \Omega_s$$

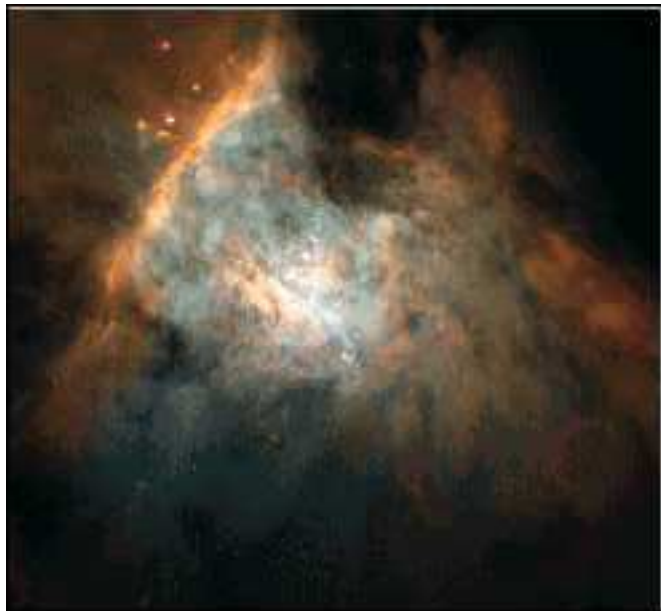
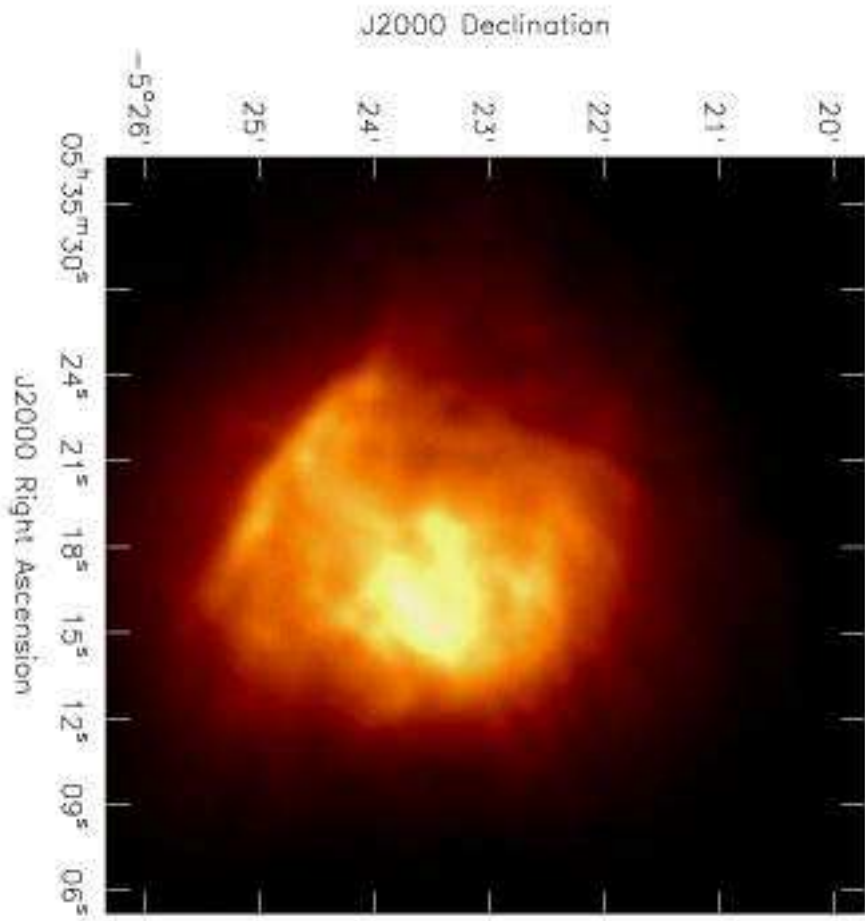
Since  $\tau \propto EM$ , EM can be derived from  $S_\nu$  and  $T_e$ !

3. **Electron Density:** if the HII region has a uniform density over a roughly spherical volume with a radius  $r$ ,

$$n_e = (EM/2r)^{1/2}$$

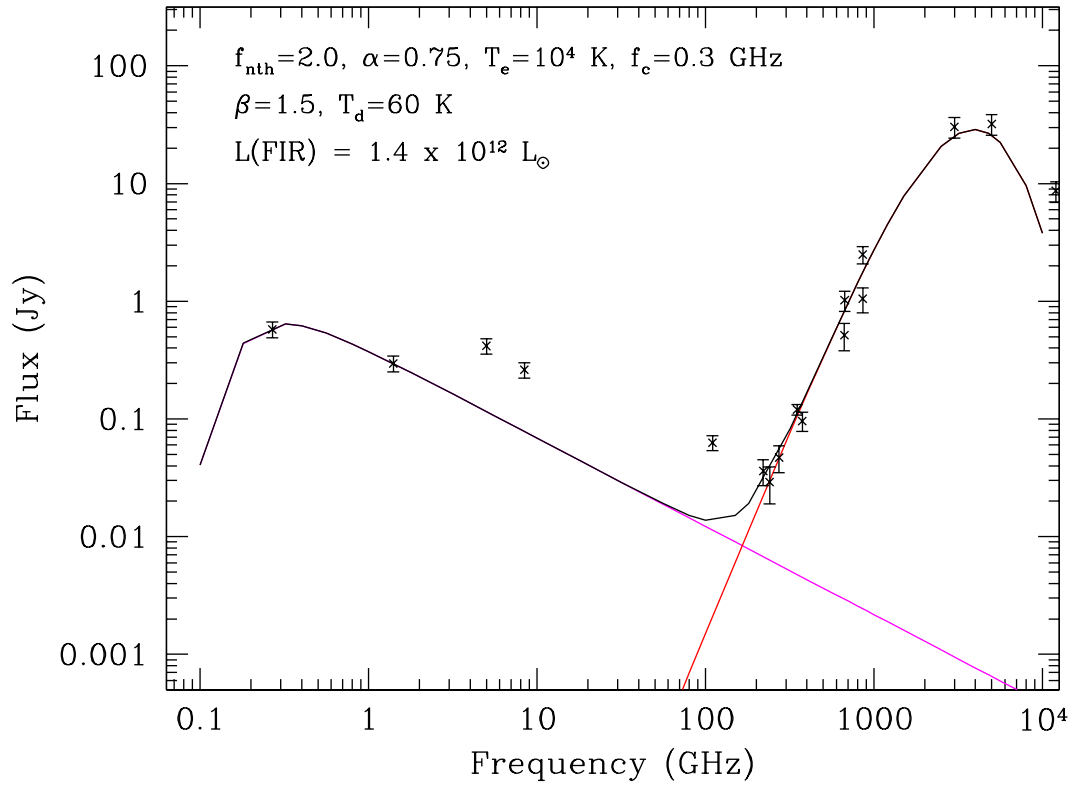
4. **Mass:** if the HII region is primarily of ionized hydrogen,

$$M = n_e \times \left(\frac{4}{3}\pi r^3\right) \times m(H)$$



# Examples of free-free Absorption:

## MRK231 SED



## Free-Free Absorption Model for PKS 1718-649

