# NON-THERMAL RADIO EMISSION

Rayleigh-Jeans Limit:

$$I_\nu(T) = \frac{2\nu^2}{c^2} kT$$

$$S_\nu(T) = \frac{2\nu^2}{c^2} kT\Omega_s$$

$$\Rightarrow T = \frac{S_\nu c^2}{2\nu^2 k\Omega_s}$$

**Q: What if $T \geq 10^5$ K? $T \geq 10^7$ K?**

# Synchrotron Radiation of a Single Electron

## Relativistic Einstein-Planck Eqn:

$$\frac{d}{dt}(\gamma m\vec{v}) = e(\vec{v} \times \vec{B})$$

If no E field, then no net force and $\gamma \equiv [1 - (\frac{v}{c})^2]^{-1/2}$ and $|\vec{v}|$ constant. Separating the velocity components along $\vec{B}$,

$$\frac{d\vec{v_\parallel}}{dt} = 0, \ \frac{d\vec{v_\perp}}{dt} = \frac{e}{\gamma m}(\vec{v_\perp} \times \vec{B})$$

$\Rightarrow \ \vec{v_\parallel} =$ constant and $\vec{v_\perp} =$ constant as well

because $|\vec{v}| =$ constant. The solution is a uniform circular motion with a constant acceleration perpendicular to $\vec{B}$ with magnitude $a_\perp = \frac{eB}{\gamma m}v_\perp$ – a helical trajectory with a constant speed along $\vec{B}$.

The angular frequency of the orbit is $w_B = \frac{eB}{\gamma m} = \frac{w_G}{\gamma}$, where $w_G = 17.6(\frac{B}{\text{Gauss}})$ MHz is the gyration frequency.

## Power Radiated:

From the Larmor formula $P(t) = \frac{2}{3}\frac{e^2\dot{v}^2}{c^3}$, $P = \frac{2e^2}{3c^2}\gamma^4 a_\perp^2$ and

$$P = \frac{2e^2}{3c^2}\gamma^4(w_B v_\perp)^2 = \frac{2e^4 v_\perp^2 B^2}{3m^2c^2}\gamma^2$$

averaging over all directions,

$$P = \frac{4}{3}\beta^2 c\sigma_T\gamma^2 U_B$$

where $\sigma_T$ is Thompson cross section and $U_B = B^2/4\pi$ is magnetic energy density.

When $\gamma \gg 1$, $\beta \sim 1$, and $P \propto \gamma^2$!!

**Relativistic Beaming:**

- Energy radiated into a narrow cone: $\theta = 2/\gamma$ radian
- Duration of the pulse: $\Delta t = \theta / w_B = \frac{2mc}{eB}$
- Pulse shortened by finite speed of light: $\Delta t' = \frac{mc}{eB\gamma^2}$
- Characteristic frequency $\nu_c = 1/\Delta t' = \frac{eB\gamma^2}{mc}$

$$\nu_c(GHz) \sim 0.016(\frac{B}{\mu G})(\frac{E}{\mathrm{GeV}})^2$$

- Maximum emission occurs at $\nu_{peak} = \frac{1}{2}\nu_c$
- Synchrotron lifetime: $\tau_s \equiv E/\dot{E}$

$$\tau_s(yrs) \sim 10^9(\frac{B}{\mu G})^{-3/2}(\frac{\nu_c}{GHz})^{-1/2}$$

[e.g. $10^8$ yr at $\nu_c = 1$ GHz and $B = 5$ $\mu$G]

** Check Condon & Ransom notes for derivations **

## Radiation from an Ensemble of Electrons

From empirical evidence, a power law distribution is reasonable for electron energy distribution

$$N(E)dE = KE^{-p}dE$$

Since the power radiated is proportional to $\gamma^2 = (\frac{E}{mc^2})^2$

$$dP \propto N(E)E^2dE \propto E^{2-p}dE$$

Most power radiated at $\nu_{peak}$, then

$$\nu \propto \gamma^2 \propto E^2 \text{ and } d\nu/dE \propto E$$

Thus, $dP \propto (\nu^{(2-p)/2})(\nu^{-1/2}d\nu) \propto \nu^{(1-p)/2}d\nu$, and since $S_\nu = dP/d\nu$

$$S_\nu \propto \nu^{(1-p)/2}$$

Commonly, spectral index $\alpha$ is defined as $S_\nu \propto \nu^{-\alpha}$, and thus $\alpha = (p-1)/2$.

Example: typically $\alpha = 0.7$ (e.g. SNR)

$$N(E) \propto E^{-2.4} \text{ !}$$

Volume emissivity:

$$\epsilon(\nu) = \int_E P(\nu, E)N(E)dE$$

$$P(\nu) = \frac{\sqrt{3}e^3 B sin\phi}{mc^2}\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)d\eta$$

where $K_{5/3}$ is the modified Bessel function of 5/3 order.

## Homogeneous Magnetic Field

$$\epsilon(\nu) = a(\alpha)K\frac{\sqrt{3}e^3}{8\pi mc^2}[\frac{3e}{4\pi m^3c^5}]^\alpha(Bsin\phi)^{\alpha+1}\nu^{-\alpha}$$

(see RW Table 9.2 for $a(\alpha)$; $a(\alpha) \sim 1.8$ for $\alpha = 0.7$)

## Random Magnetic Field

For random orientation of B field, the $(Bsin\phi)^{\alpha+1}$ term needs to be averaged over all angles, and

$$\epsilon(\nu) = b(\alpha)K\frac{e^3}{mc^2}[\frac{3e}{4\pi m^3c^5}]^\alpha B^{\alpha+1}\nu^{-\alpha}$$

(see RW Table 9.2 for $b(\alpha)$; $b(\alpha) \sim 0.087$ for $\alpha = 0.7$)

[Note that polarization averages out to zero in the random magnetic field case (see below).]

# Polarization of Synchrotron Radiation

Radiation field of highly relativistic electrons orbiting in a homogeneous B field is strongly beam by the energy $\gamma$ of the electron. It is detected only when beamed toward the observer, and the instantaneous radiation is generally elliptically polarized. Since the PA of the polarization ellipse is rotating with electron, the time averaged polarization is linear (see RW 9.8 and RL 6.4 for more details).

For particles with a power law distribution of energies, the degree of polarization is

$$\Pi = \frac{p+1}{p+7/3}$$

(for $\alpha = 0.7$, linear polarization may be as large as 70% !)

## Energy Requirements

Energy in a nonthermal source is present in two different forms: $W_p$ (particle energy) and $W_B$ (magnetic energy).

$$W_{total} = W_{part} + W_{mag} = V(u_p + u_B)$$

$$u_p = \eta u_e = \eta K \int_{E_{min}}^{E_{max}} E^{1-p} dE$$

$$u_B = \frac{B^2}{8\pi}$$

Since radiated power peaks strongly near $\nu_c$, substitute $\nu = \frac{3eB}{2m^3c^5}E^2$ and $\alpha = \frac{p-1}{2}$

$$u_p = K \times G \times B^{\alpha-1/2}$$

where $G = \frac{\eta}{1-2\alpha}(\frac{e}{m^3c^5})^{\alpha-1/2}(\nu_{max}^{1/2-\alpha} - \nu_{min}^{1/2-\alpha})$

$$\Rightarrow W_{tot} = V \times (K \times G \times B^{\alpha-1/2} + \frac{B^2}{8\pi})$$

For a source with a volume V at a distance R, observed flux density is

$$S_\nu = KHVB^{\alpha+1}\nu^{-\alpha}/R^2$$

where $H = b(\alpha)\frac{e^3}{mc^2}(\frac{3e}{4\pi m^3c^5})^\alpha$. Substituting this back into $W_{tot}$, one gets

$$W_{tot} = \frac{G}{H}R^2(S_\nu \nu^\alpha)B^{-3/2} + \frac{VB^2}{8\pi}$$

$W_{tot}$ has the minimum value when

$$B_{eq} = (6\pi \frac{GR^2}{HV} S_\nu \nu^\alpha)^{2/7}$$

$$W_{tot} = \frac{7}{6}(6\pi)^{-3/7}(\frac{G\nu^\alpha}{H} S_\nu)^{4/7} R^{8/7} V^{3/7}$$

This equation gives an estimate of the minimum energy requirements of a synchrotron source.