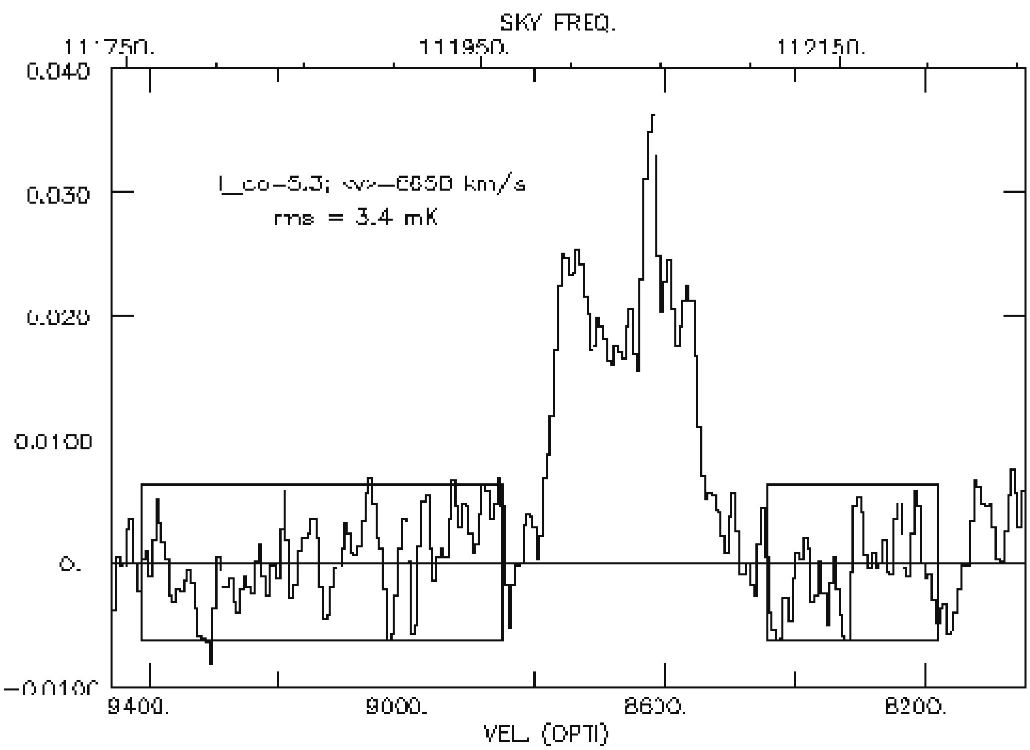
SPECTROSCOPY I

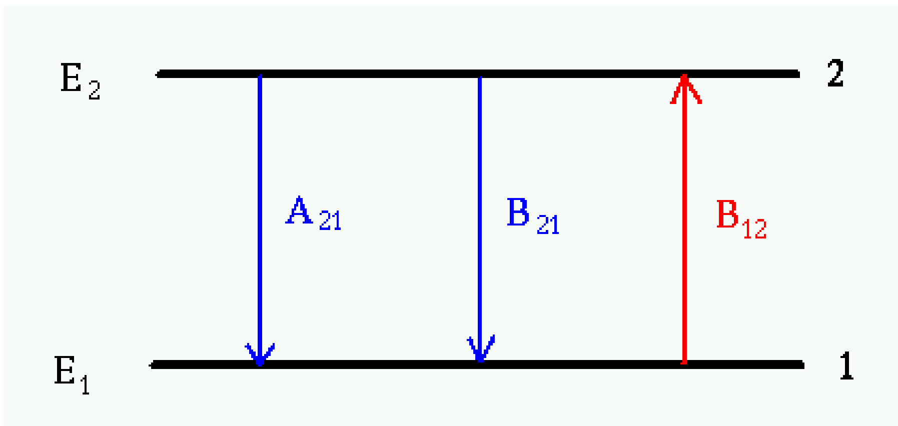
Advantages at Radio Wavelengths

- ν or λ known with high accuracy
- high spectral resolution ($[\frac{\Delta\nu}{\nu}]^{-1} > 10^{6-7}$)
- $\nu = \frac{kT}{h} = 20.8$ T GHz



Spectroscopy Fundamentals

Einstein Coefficients



- $E_2 - E_1 = h\nu$
- radiation density $\bar{U} \equiv 4\pi\bar{I}/c$
- A_{21} = probability for emitting a photon spontaneously
- $B_{21}\bar{U}$ = probability for stimulated emission
- $B_{12}\bar{U}$ = probability of the absorption of a photon

$$n_2 A_{21} + n_2 B_{21}\bar{U} = n_1 B_{12}\bar{U}$$

Thermal Equilibrium

Boltzmann distribution:

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{kT}}$$

where g_1 and g_2 are statistical weights.

$$\bar{U} = \frac{A_{21}}{\frac{n_1}{n_2} B_{12} - B_{21}} = \frac{A_{21}}{\frac{g_1}{g_2} e^{+\frac{h\nu}{kT}} B_{12} - B_{21}}$$

From the Planck Function,

$$\bar{U} = \frac{4\pi}{c} B_\nu(T) = \frac{8\pi h\nu^3 / c^3}{e^{+\frac{h\nu}{kT}} - 1}$$

Therefore,

$$\boxed{\begin{array}{c} \dfrac{g_2}{g_1} = \dfrac{B_{12}}{B_{21}} \\ A_{21} = \dfrac{8\pi h\nu^3}{c^3} B_{21} \end{array}}$$

Radiative Transfer

- spontaneous emission:

$$dE_e(\nu) = h\nu n_2 A_{21} \varphi_e(\nu) dV \frac{d\Omega}{4\pi} d\nu dt$$

- stimulated emission:

$$dE_s(\nu) = h\nu n_2 B_{21} \frac{4\pi}{c} I_\nu \varphi_s(\nu) dV \frac{d\Omega}{4\pi} d\nu dt$$

- absorption:

$$dE_a(\nu) = h\nu n_1 B_{12} \frac{4\pi}{c} I_\nu \varphi_a(\nu) dV \frac{d\Omega}{4\pi} d\nu dt$$

$$dI_\nu d\Omega d\sigma d\nu dt = dE_e(\nu) + dE_s(\nu) - dE_a(\nu)$$

$$\boxed{\frac{dI_\nu}{ds} = -\frac{h\nu}{c}(n_1 B_{12} - n_2 B_{21}) I_\nu \varphi(\nu) + \frac{h\nu}{4\pi} n_2 A_{21} \varphi(\nu)}$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + \varepsilon_\nu$$

$$\boxed{\begin{aligned} \kappa_\nu &= \frac{h\nu}{c} n_1 B_{12} (1 - \frac{g_1 n_2}{g_2 n_1}) \varphi(\nu) \\ \varepsilon_\nu &= \frac{h\nu}{4\pi} n_2 A_{21} \varphi(\nu) \end{aligned}}$$

In general,

$$n_1(C_{12} + B_{12}\bar{U}) = n_2(A_{21} + B_{21}\bar{U} + C_{21})$$

$$C_{ij} = \int_0^\infty \sigma_{ij}(v) v f(v) dv$$

where σ_{ij} is the collision cross section and $f(v)$ is the velocity distribution function of the particles.

Simple Solutions

1. LTE case: $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{kT}}$

2. Non-LTE case: $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{kT_{ex}}}$

• If radiation dominates,

$$n_1 B_{12}\bar{U} = n_2(A_{21} + B_{21}\bar{U})$$

$$\bar{U} = \frac{4\pi}{c}\bar{I} = \frac{8\pi h\nu^3}{c^3}[exp(\frac{h\nu}{kT_b}) - 1]^{-1}$$

Re-arraning first eq. and using $A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21}$

$$\frac{n_2}{n_1} = \frac{B_{12}}{B_{21}} \frac{\bar{I}}{\frac{2h\nu^3}{c^2} + \bar{I}} = \frac{g_2}{g_1} e^{-\frac{h\nu}{kT_b}}$$

where T_b is brightness temperature.

• If collision dominates,

$$\frac{C_{12}}{C_{21}} = \frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{kT_K}}$$

where T_K is "kinetic temperature".

In general, the excitation temperature is given by

$$\boxed{T_{ex} = T_K \frac{T_b A_{21} + \frac{h\nu}{k} C_{21}}{T_K A_{21} + \frac{h\nu}{k} C_{21}}}$$

- radiation dominates: $T_{ex} \rightarrow T_b$
- collision dominates: $T_{ex} \rightarrow T_K$
- critical density: $A_{21} \sim C_{21} \sim n^* < \sigma v >$

21cm Neutral Hydrogen (HI) Line

The 21cm HI line is the transition between the hyperfine structure levels $1^2S_{1/2}, F = 0$ and $F = 1$ of neutral hydrogen – energy difference from the spin of the proton and the electron.

- $\nu_{10} = 1.420405752 \times 10^9$ Hz
- $A_{10} = 2.868887 \times 10^{-15}$ s^{-1}
- mean half-life: $t_{1/2} \sim 1/A_{10} = 3.49 \times 10^{14}$ s $=$ 1.11×10^7 yr

  $\Rightarrow$ thus collisionally excited!
- "spin temperature" T_s: $\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\frac{h\nu}{kT_s}}$
- $\frac{n_1}{n_0} = \frac{g_1}{g_0} = 3$ for $T_s \gg \frac{h\nu}{k} = 0.0682$ K
- $T_s \sim 100K$, typically in cold ISM and can be much higher.
- $\int \tau dv = -\int \kappa_\nu ds = 5.49 \times 10^{-19} T_s^{-1} \int n_H(s) ds$

$$N_H = 1.82 \times 10^{18} T_s \int \tau(v) dv \text{ cm}^{-2}$$

HI in External Galaxies

- $M_{HI} = 530 D_L^2 \theta^2 \int T_{mb} dv \ M_\odot$ or
  $M_{HI} = 2.36 \times 10^5 D_L^2 \int S_v(Jy) dv \ M_\odot$
  where D_L is luminosity distance in Mpc, θ is beam size in arcsec, and v is velocity in km/s.
- $D_{HI} = (1-3) \times D_{25}$
- virial mass: from $\bar{v^2} = \frac{GM}{2R}$

$$M_{vir} = 250 (\frac{\Delta v_{1/2}}{km/s})^2 (\frac{R}{pc}) \ M_\odot$$