Ordinary Differential Equations

Computational Physics

Ordinary Differential Equations
First Order ODE's

Outline

- Ordinary Differential Equations
- Numerical Approximations
- Iterative Method of Solution
- The Euler Method

Ordinary Differential Equations

$$\frac{dN}{dt} = -kN$$

First Order

Radioactive Decay

$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

Second Order

Projectile Motion

Numerical Approximations
Taylor Series Expansion

$$N(\Delta t) = N(0) + \frac{dN}{dt}\Delta t + \frac{1}{2}\frac{d^2N}{dt^2}\Delta t^2 + \cdots$$

$$N(\Delta t) \approx N(0) + \frac{dN}{dt}\Delta t$$

We can find N at some future time with knowledge of:

(1) the current value of N;
and
(2) the first time derivative of N.

Numerical Approximations

Approximation of 1st Derivative

$$\frac{dN}{dt} \equiv \lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{\Delta t}$$

$$\frac{dN}{dt} \approx \frac{N(t+\Delta t) - N(t)}{\Delta t}$$

$$N(t+\Delta t) \approx N(t) + \frac{dN}{dt}\Delta t$$

We can find N at some future time with knowledge of:

(1) the current value of N; and

(2) the first time derivative of N.

A simple problem
Radioactive Decay

Differential Equation:

$$\frac{dN}{dt} = -kN$$

The rate of change in the number of radioactive atoms (*N*) is proportional to the number.

k is the constant of proportionality.

Initial Conditions:

Unique solution to a First Order DEQ requires specification of the solution at some time, commonly taken as the “initial time” t=0. In this case, we would specify the number of atoms at t=0.

Iterative Method of Solution

First Order ODE

Consider array of times for calculation:

$$t_0 , t_1 , t_2 , t_3 , \ldots , t_M$$

$$t_{i+1} = t_i + \Delta t$$

Differential Equation

$$\frac{dN}{dt} = -kN$$

Corresponding array of solution, *N*

$$N_0 , N_1 , N_2 , N_3 , \ldots , N_M$$

with N_0 known at t_0 from initial conditions.

Euler Algorithm

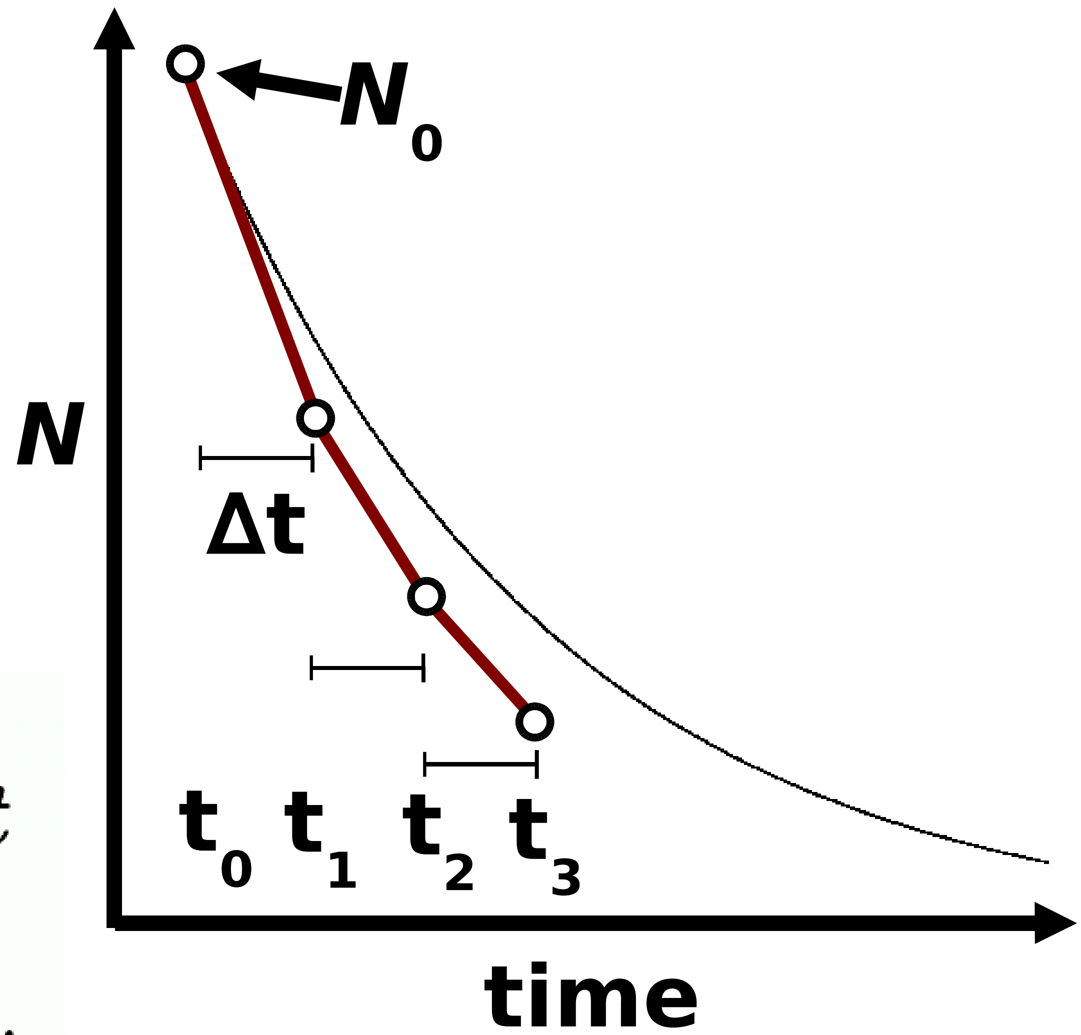
Initialize at initial time:

N_0 = initial value

Start from initial value and iterate for all future times:

$$N_{i+1} \approx N_i + \frac{dN}{dt}\Delta t$$

$$N_{i+1} \approx N_i - kN_i\Delta t$$



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% Example: Euler Method for Radioactive Decay
clear
k = 0.05;          % decay constant
N0 = 1000;         % initial value
dt = 1;            % step size
t = 0:dt:100;      % array of times for calculation

N = zeros(length(t),1);   % define solution array

N(1) = N0;
for i=1:length(t)-1
   N(i+1) = N(i) - k*N(i)*dt;
end

% make some nice graphs
plot(t,N,'md')
hold on
plot(t,N0*exp(-k*t),'k')
hold off
xlabel('t');
ylabel('N');
legend('Euler Method','Exact')
```