Ordinary Differential Equations

Computational Physics

Ordinary Differential Equations
Second Order ODE's

Outline

- Second Order Ordinary Differential Equations
- General Method of Solution
- Euler's Method

Ordinary Differential Equations

$$\frac{dN}{dt} = -kN$$

First Order

Radioactive Decay

$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

Second Order

Projectile Motion

Method for Second Order DEQ's

$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

Write

Second Order Equation

as

$$\frac{dv}{dt} = \frac{F}{m}$$

$$\frac{dx}{dt} = v$$

two simultaneous
First Order Equations

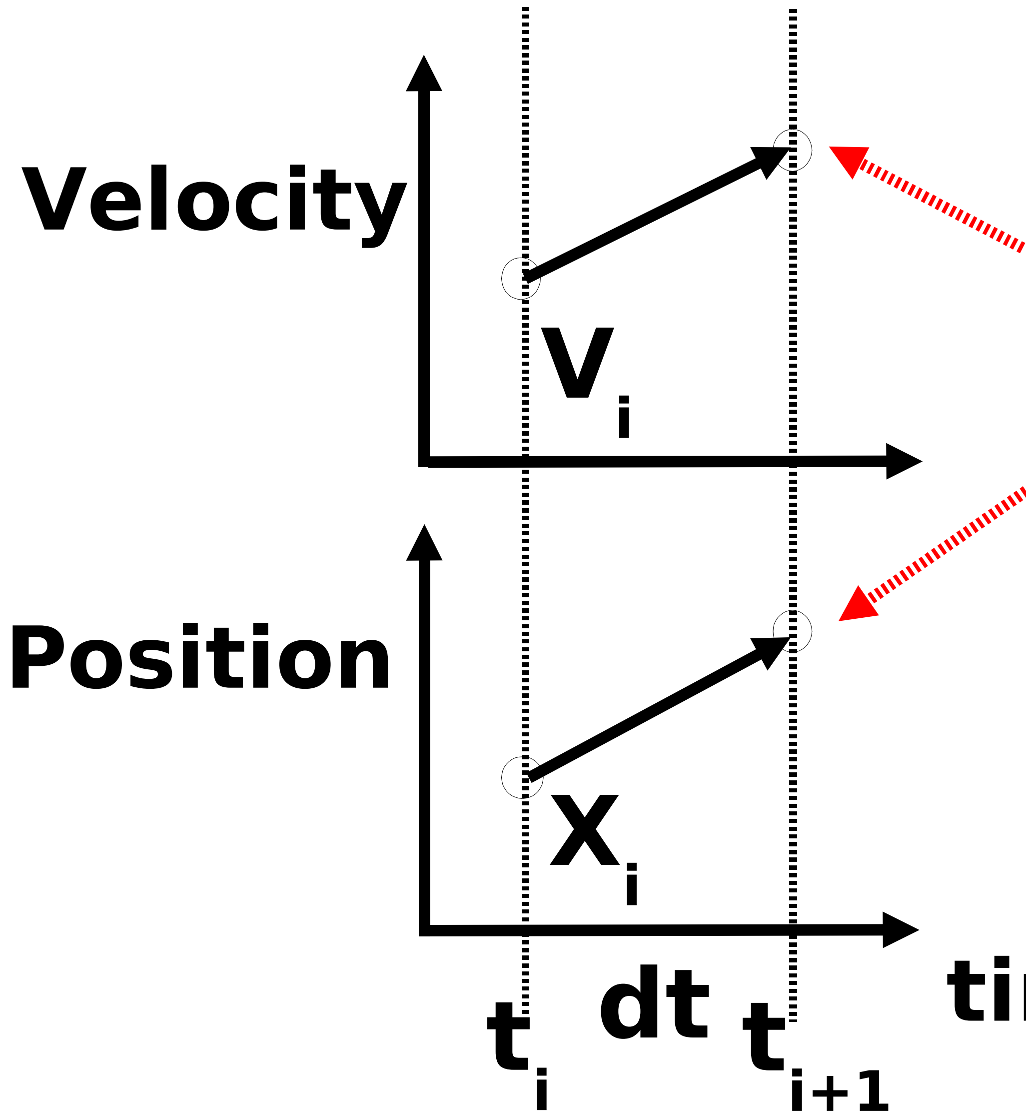
Second Order DEQ

Initial Conditions

- First Order DEQ required specification of solution at initial time.
  - e.g. value of N at t=0 in radioactive decay problem.
- Second Order DEQ requires initial values for *each* First Order DEQ.
  - velocity at initial time (t=0)
  - position at initial time (t=0)

Second Order DEQ

Euler Method



$$V_{i+1} = V_i + F/m \, dt$$

$$X_{i+1} = X_i + V_i \, dt$$

Solve Simultaneously

Second Order DEQ

Euler Method

Formula to obtain solution at next time step:

$$v_{i+1} = v_i + \frac{F_i}{m} dt \qquad v \text{ solution}$$

$$x_{i+1} = x_i + v_i\, dt \qquad x \text{ solution}$$

where:

F_i = force at time step i

v_i = velocity at timestep i

x_i = position at timestep i

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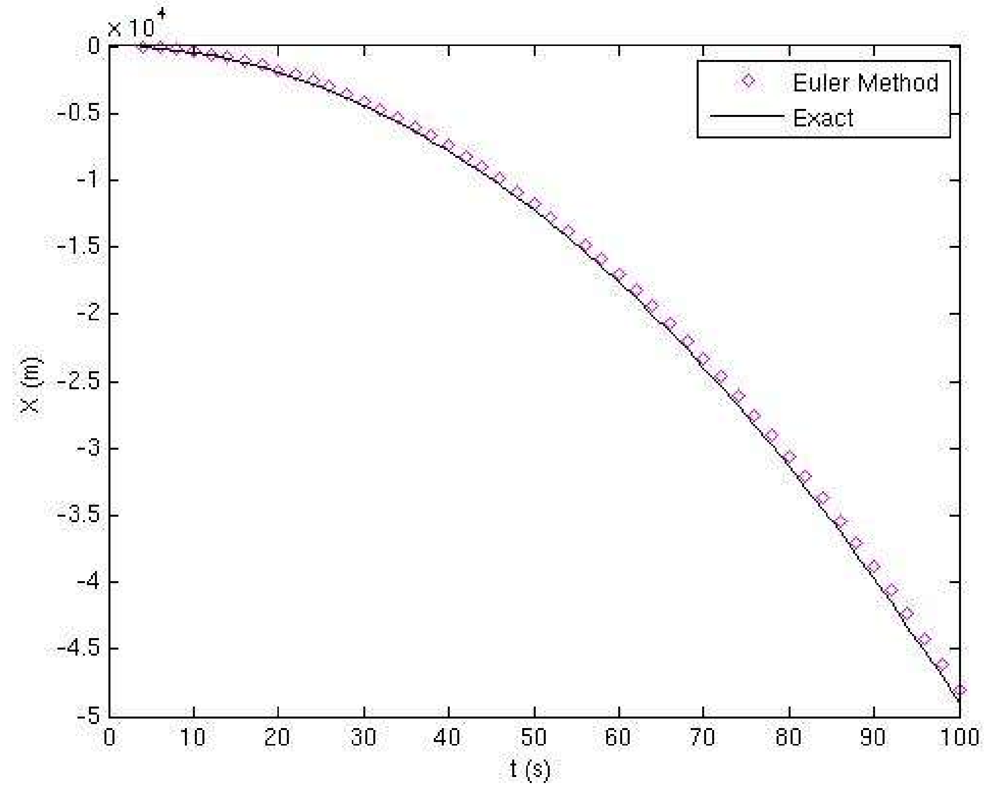
% Example: Falling Ball
clear
g = 9.8;          % gravitational acceleration
X0 = 0;           % initial value: position
V0 = 0;           % initial value: velocity
dt = 2;           % step size
t = 0:dt:100;     % array of times for calculation

X = zeros(length(t),1);   % define solution arrays
V = zeros(length(t),1);
X(1) = X0;
V(1) = V0;
for i=1:length(t)-1
   V(i+1) = V(i) – g*dt;
   X(i+1) = X(i) + V(i)*dt;
end

% make some nice graphs
plot(t,X,'md')
hold on
plot(t,X0+V0*t-0.5*g*t.*t,'k')
hold off
xlabel('t (s)');
ylabel('X (m)');
legend('Euler Method','Exact')
```



Euler Method

Consider Single Step with Falling Ball

$$v(dt) = v_0 - g\ dt$$

$$v_{true}(dt) = v_0 - g\ dt$$

Velocity equation is exact

$$x(dt) = x_0 + v_0\ dt$$

$$x_{true}(dt) = x_0 + v_0\ dt - \frac{1}{2}\ g\ dt^2$$

Position equation missing quadratic term

Initial E: $E_0 = mgx_0 + \frac{1}{2}mv_0^2$

E after one step: $E(dt) = mgx_0 + \frac{1}{2}mv_0^2 + \frac{1}{2}mg^2\ dt^2$

Energy is not conserved!