

Project 1

Computational Physics

Project 1

Radiative Transfer in a Planetary
Atmosphere

Outline

- Energy Balance and Effective Temperature
- Radiative Transfer in Planetary Atmosphere
- A Random Walk Model

Energy Balance

- Temperature of a Planet is determined by balance between
 - Sunlight absorbed
 - Blackbody radiation from planet to space
- Sunlight Absorbed: $E_{abs} = (1 - A) S \pi R^2$
 - S: Solar Constant (energy sec⁻¹ area⁻¹)
 - A: Albedo (fraction of sunlight reflected)
 - R: Planetary Radius
- Blackbody Radiation: $E_{rad} = 4\pi R^2 \sigma T^4$
 - T: Blackbody Temperature
 - σ : Stefan-Boltzman Constant

Effective Temperature

- T_{eff} is temperature of blackbody which radiates amount of energy equal to energy absorbed from Sun.

$$(1 - A) S \pi R^2 = 4\pi R^2 \sigma T_{eff}^4$$

$$T_{eff} = \left(\frac{(1 - A)S}{4\sigma} \right)^{1/4}$$

Planetary Temperatures

PLANET	Distance AU	S (J/m ² /s)	A	Teff K	P Surface (bars)	T Surface K
Mercury	0.39	9127	0.06	441	0	100-700
Venus	0.72	2615	0.71	240	80	700
Earth	1.00	1367	0.33	252	1	288
Moon	1.00	1367	0.07	274	0	100-400
Mars	1.52	589	0.17	215	0.01	220

NOTE TO TABLE: ALL VALUES ARE APPROXIMATE

Planetary Surface Temperatures can exceed the Effective Temperature by quite a lot.

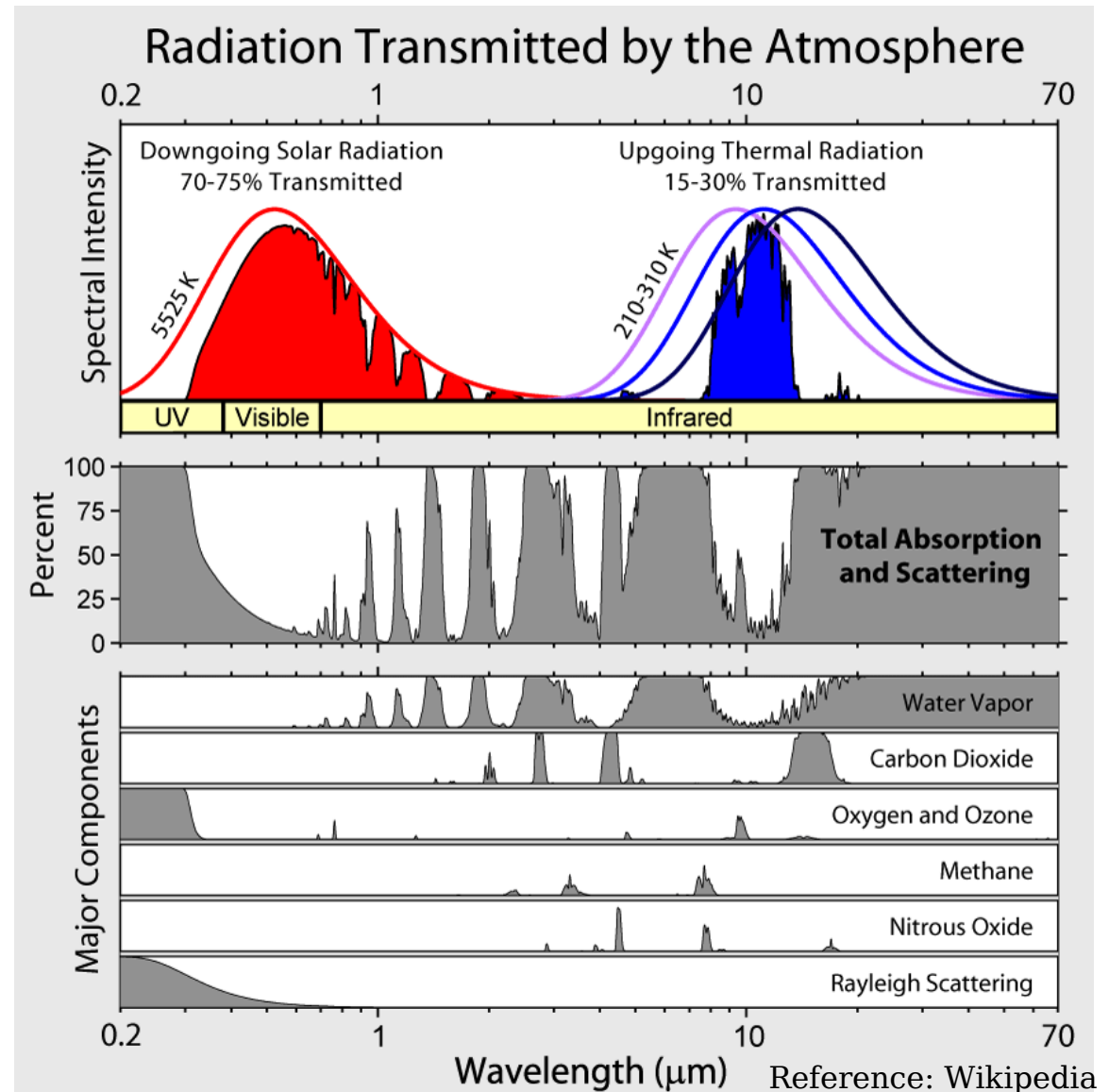
This is due to the presence of the atmosphere and warming of the surface by the “*Greenhouse Effect*”.

Radiation in the Atmosphere

Absorption of Sunlight *heats* the Planet. Sunlight occurs mostly in visible part of the spectrum.

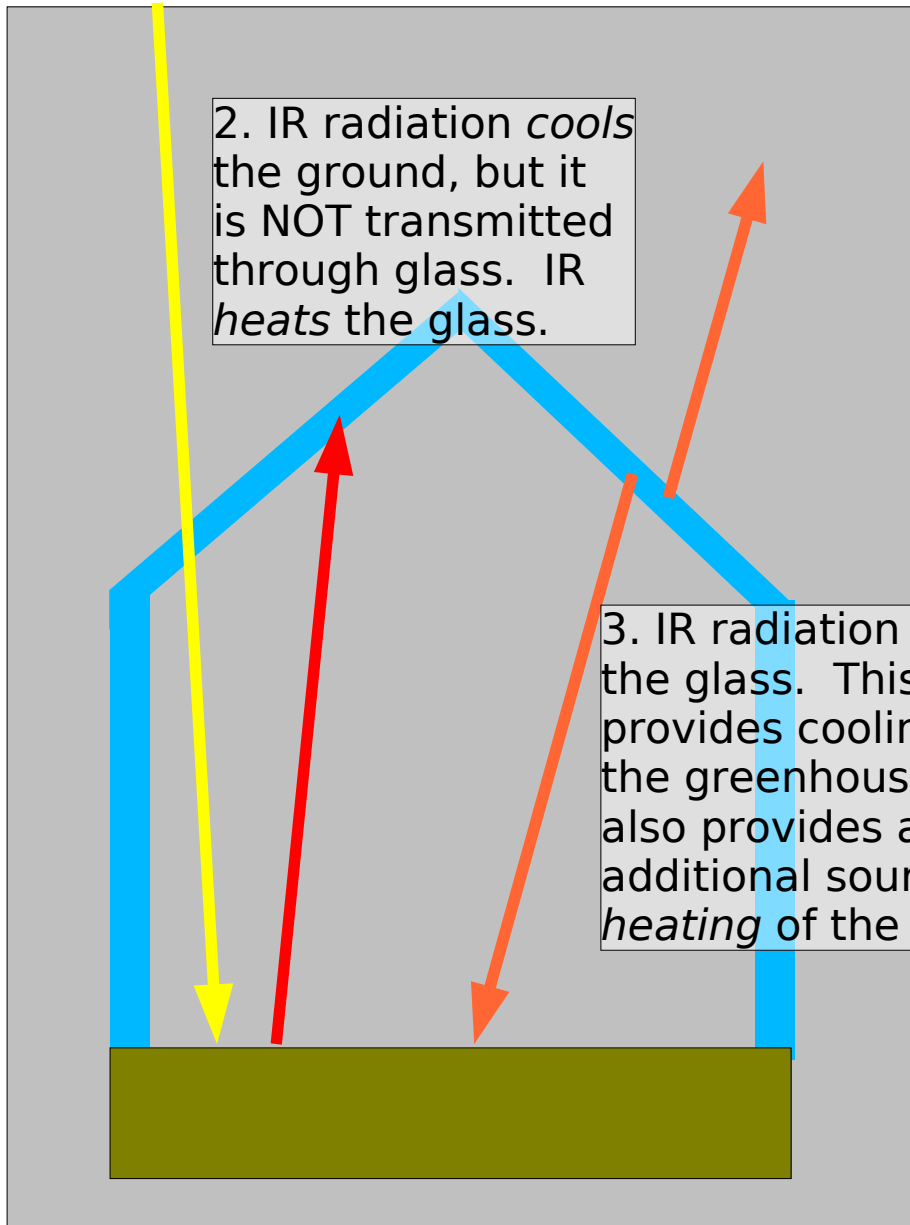
Blackbody Radiation *cools* the Planet. This occurs in the infrared ... at a *different* wavelength than absorption of sunlight.

Different wavelengths means different behavior of Atmosphere: mostly clear in visible; mostly opaque in infrared.



Radiative Transfer in a Greenhouse

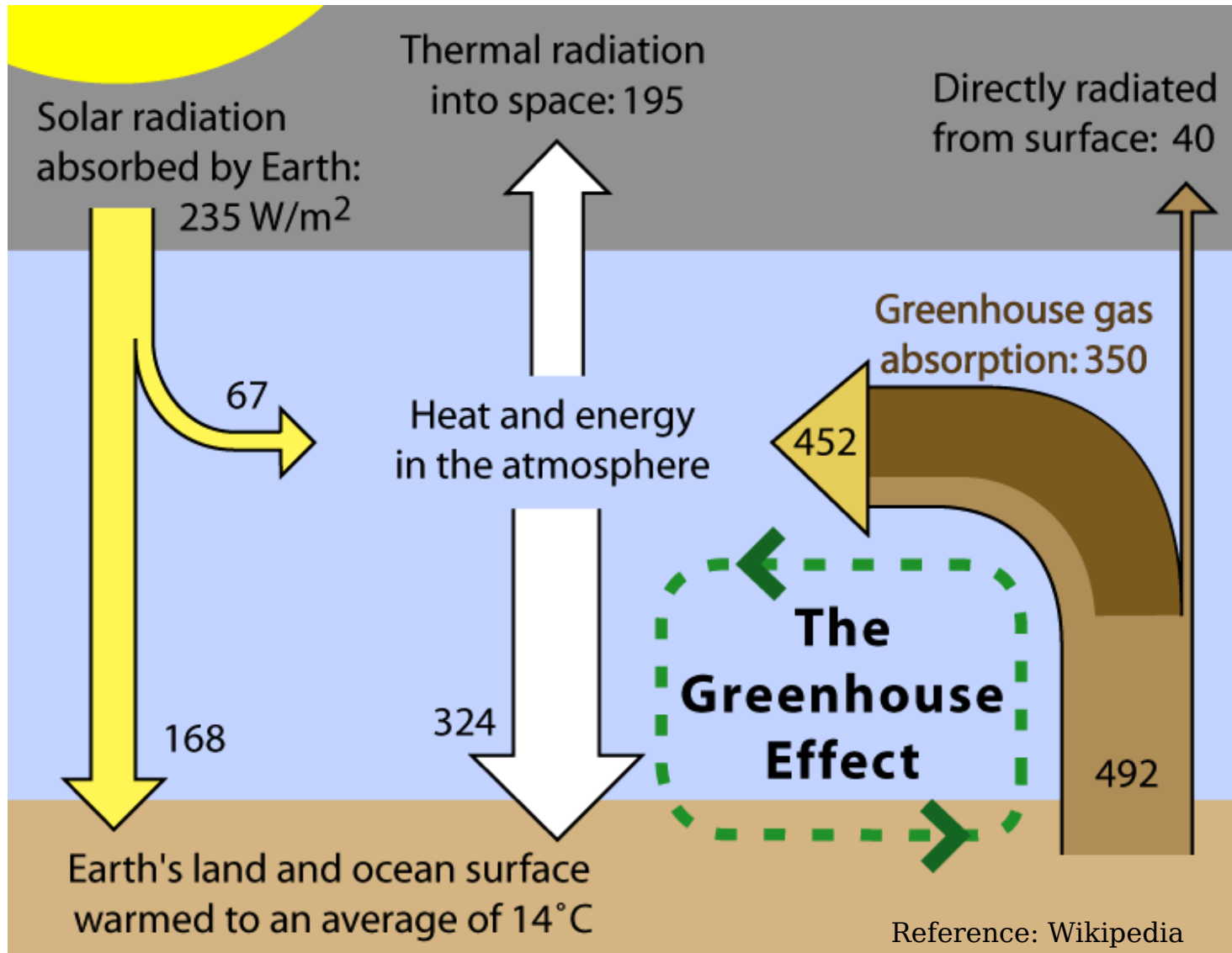
1. Sunlight is transmitted through glass and heats ground directly



The behavior of radiation in a planetary atmosphere is often compared to behavior of an idealized greenhouse.

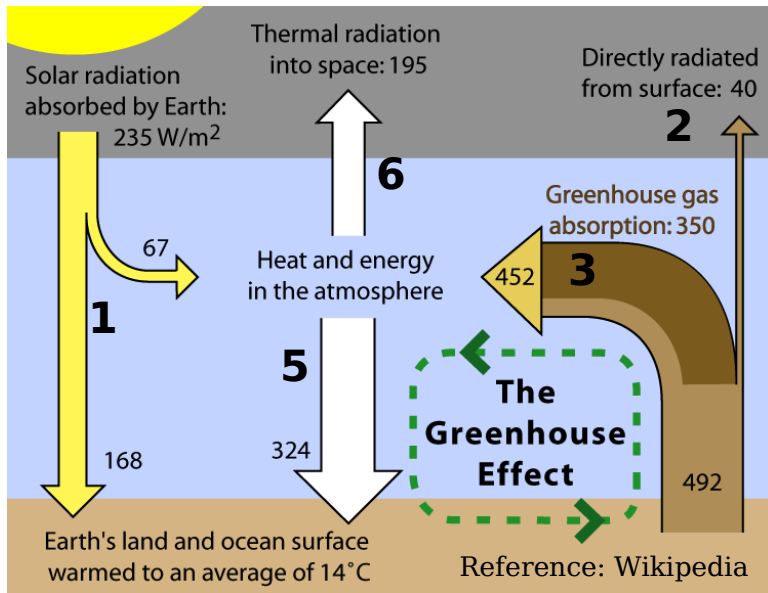
The key point is that the atmosphere plays the role of the glass in this model and the transfer of energy through the atmosphere from the ground is critical to determining the ground temperature.

Energy Balance in Earth's Atmosphere



Energy Balance (annotated)

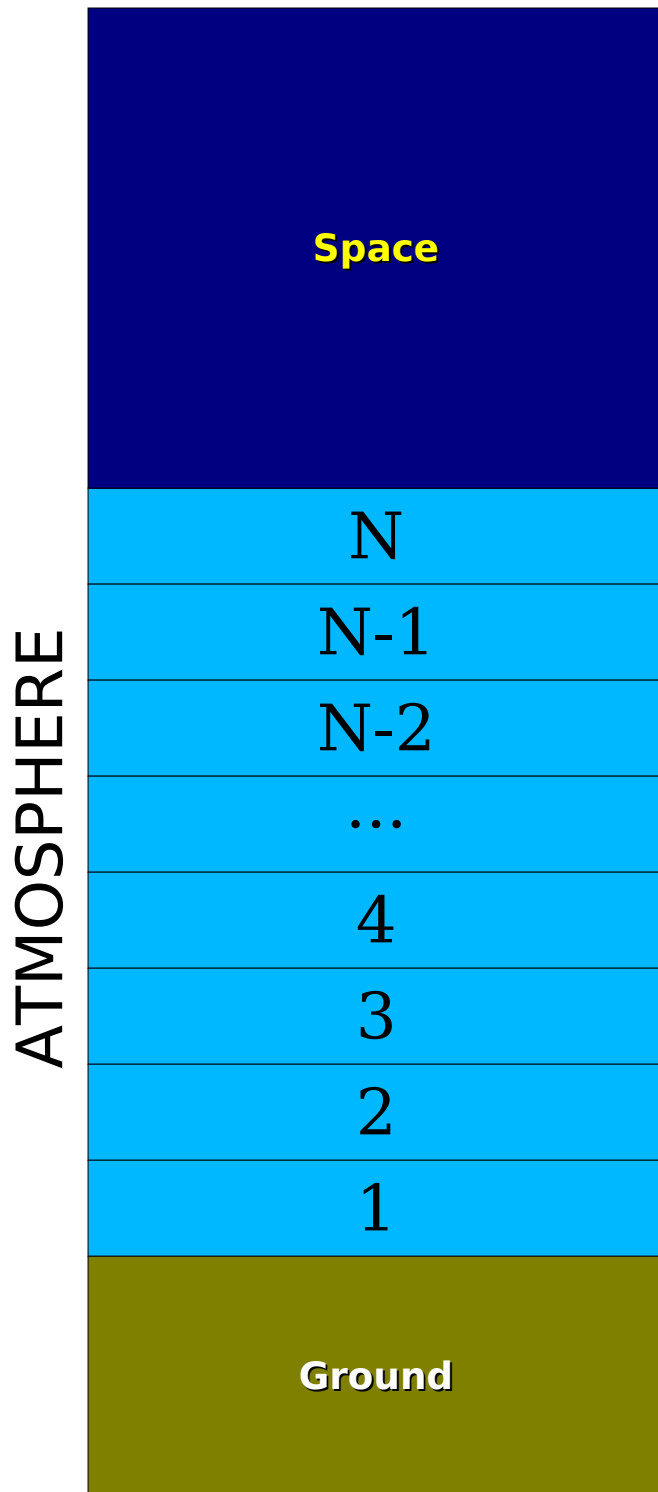
Total absorbed by Planet equals total emitted back to space



Total absorbed by surface from Sun and Atmosphere equals total emitted from the surface.

1. Sunlight heats ground directly. (Some sunlight directly absorbed by atmosphere too.)
2. Some IR cools ground and escapes directly back to space.
3. Some IR cools ground but is absorbed by the atmosphere.
4. Atmosphere emits IR which is absorbed elsewhere in atmosphere. Net effect is transfer of energy within atmosphere.
5. Atmosphere emits IR radiation which heats ground.
6. Atmosphere emits IR to space, helping to cool Planet.

Random Walk Model



Divide Atmosphere into N Layers*.

Photon emitted by ground moving upward.

At each layer, there is a chance that the photon will be absorbed*.

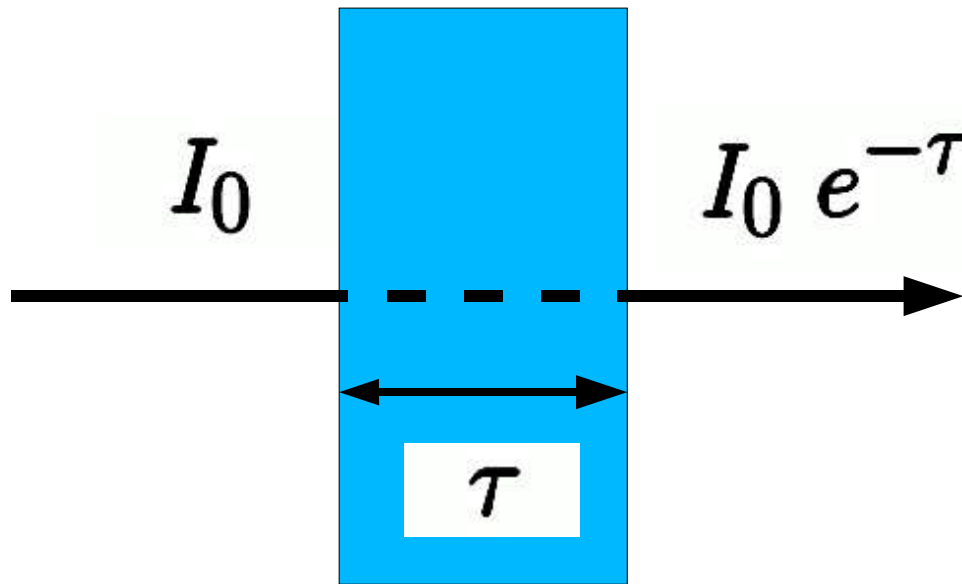
When absorbed, the energy is reemitted as a new photon going either up or down.

When NOT absorbed, photon keeps going in original direction.

We follow photon until it escapes to space or it is absorbed by the ground.

* see next slides

Probability of Absorption of Light by Matter



I_0 : Incident Intensity
 $I_0 e^{-\tau}$: Transmitted Intensity
 τ : Optical Depth

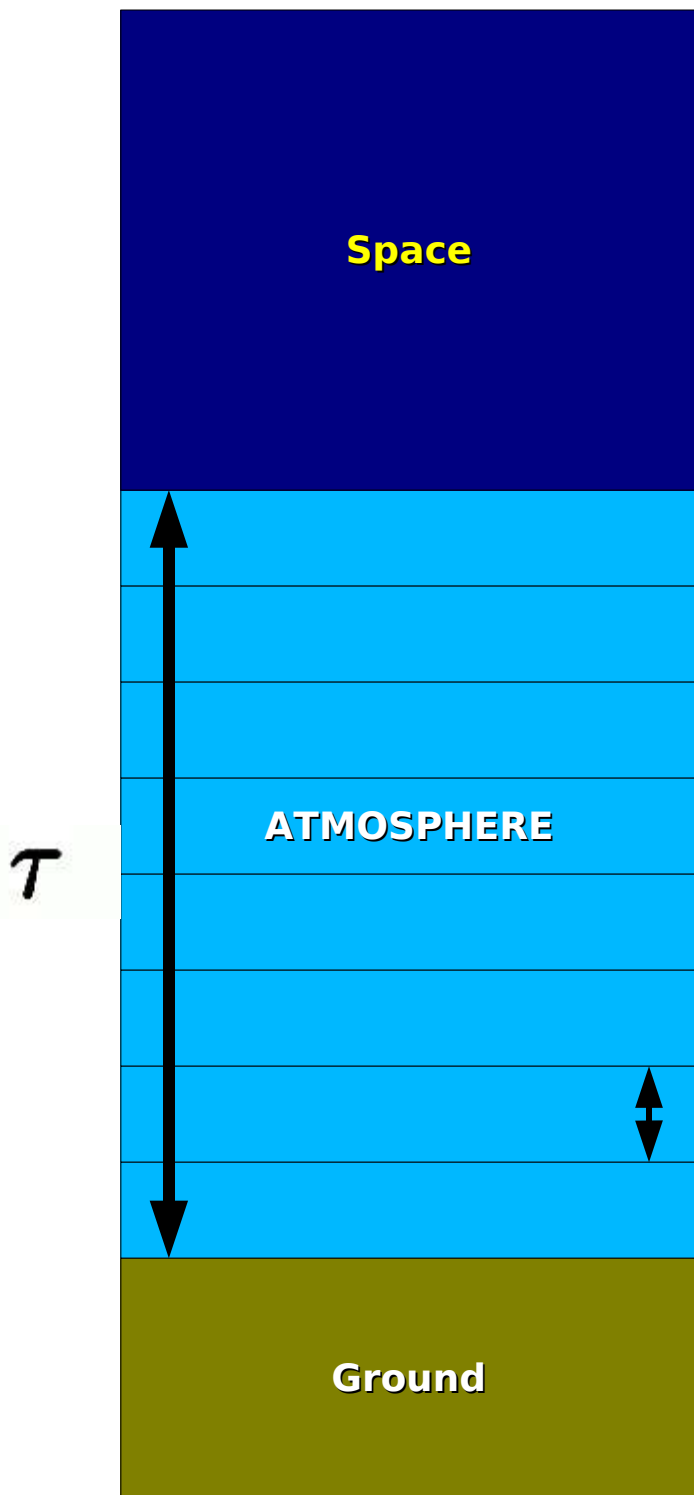
$$\textit{Fraction Absorbed} = 1 - e^{-\tau}$$

$$e^{-d\tau} \sim 1 - d\tau$$

$$\textit{Fraction Absorbed} \sim d\tau$$

} For *small* optical depth limit:
 $d\tau \ll 1$

Probability of Absorption in a Layer

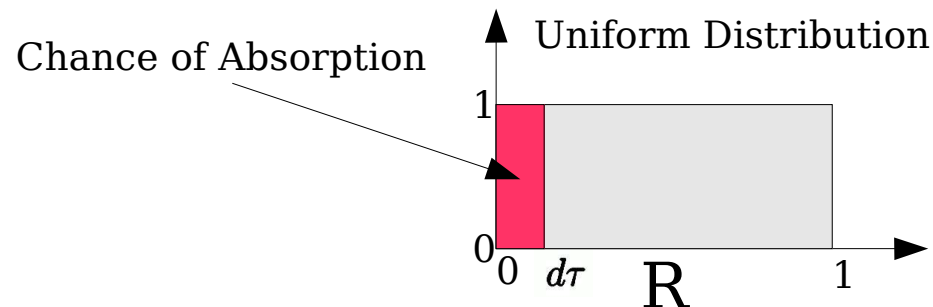


Atmosphere total optical depth: τ

Divide atmosphere into N layers with optical depth: $d\tau$

Probability of absorption by any single layer: $d\tau$

PROVIDED THAT: $d\tau \ll 1$



Results to Track

- P: Number of trial photons (user input)

The number of random walks

- P_E : Number of trial photons that escape

$P_E \propto$ Total Cooling Rate for the planet.

- P_A : Number of trial photons that are reabsorbed at the ground

$P_A/P_E \propto$ Rate of Surface Heating by the Atmosphere compared to Total Cooling Rate

- $P_L(i)$: Number of times each layer “i” absorbs a photon during the simulation

$P_L(i)/P_E \propto$ Rate of Heating of layer “i” compared to Total Cooling Rate

Relation of Results to Reality

- Normalize all Results by number of escaped photons.

$P_E \propto$ Planetary Cooling Rate: σT_{eff}^4

- Derive Ground Heating Rate relative to Planetary Cooling Rate

heating by Sun: 1 heating by Atmosphere: P_A/P_E

Net: $(1 + P_A/P_E)$

- Derive Atmospheric Heating Rate relative to Planetary Cooling Rate

Heating Rate of Layer "i" = $P_L(i)/P_E$

Relation of Results to Reality (continued)

- Derive actual temperatures by considering that radiative heating and cooling must balance:

The diagram illustrates radiative balance for two components: the ground and a layer 'i'. On the left, a brown rectangle represents the ground at temperature T_g . An upward arrow from the ground is labeled σT_g^4 . Below it, a blue rectangle represents layer 'i' at temperature T_i . Two arrows originate from layer 'i': one pointing up labeled $d\tau \sigma T_i^4$ and one pointing down labeled $d\tau \sigma T_i^4$.

$$\sigma T_g^4 = \left(1 + \frac{P_A}{P_E}\right) \sigma T_{eff}^4$$

Ground Temperature

$$2 d\tau \sigma T_i^4 = \frac{P_L(i)}{P_E} \sigma T_{eff}^4$$

$$\sigma T_i^4 = \frac{P_L(i)}{2P_E d\tau} \sigma T_{eff}^4$$

Temperature of Layer "i"

Major Assumptions

- Photons move only up and down

The “two stream” approximation.

- We ignore wavelength dependence of absorption in real atmosphere and use “average” photons

The “gray atmosphere” approximation.

- Only absorption and reemission of photons is allowed, no scattering.

Emission according to Blackbody Laws

- Radiative Heating and Cooling in Balance

Radiative Equilibrium