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The linearity of the Wesenheit function for the Large Magellanic Cloud Cepheids

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ABSTRACT

There is strong evidence that the period-luminosity (PL) relation for the Large Magellanic Cloud (LMC) Cepheids shows a break at a period around 10 days. Since the LMC PL relation is extensively used in distance scale studies, the non-linearity of the LMC PL relation may affect the results based on this LMC calibrated relation. In this paper we show that this problem can be remedied by using the Wesenheit function in obtaining Cepheid distances. This is because the Wesenheit function is linear although recent data suggests that the PL and the period-colour (PC) relations that make up the Wesenheit function are not. We test the linearity of the Wesenheit function and find strong evidence that the LMC Wesenheit function is indeed linear. We discuss this result in the context of distance scale applications. We also compare the distance moduli obtained from $\mu_0 = \mu_V - R(\mu_V - \mu_I)$ (equivalent to Wesenheit functions) constructed with the linear and the broken LMC PL relations, and find that the maximum difference in distance moduli is ~ 0.07mag., with a typical value around ~ 0.03mag. We apply the linear and broken PL relations to IC 4182, and find that the distance moduli from either approach agrees well and are consistent with each other, with a difference of ~ 0.03mag. or smaller. Hence, the broken LMC PL relation does not seriously affect current distance scale applications. We also discuss the random error calculated with equation $\mu_0 = \mu_V - R(\mu_V - \mu_I)$, and show that there is a correlation term exists from the calculation of the random error. The calculated random error will be larger if this correlation term is ignored.

Key words: Cepheids – Distance Scale

1 INTRODUCTION

The Cepheid period-luminosity (PL) relation plays a major role in distance scale studies, which can ultimately be used to determine the Hubble constant. The calibrating PL relation currently used is based mainly on the Large Magellanic Cloud (LMC) Cepheids, as applied by the H_0 Key Project team (Freedman et al. 2001) as well as in other studies (e.g., Saha et al. 2001; Kanbur et al. 2003). The Cepheid PL relation has long been considered to be a linear function of $\log(P)$ within the range of $\log(P) \sim 0.3$ to $\log(P) \sim 2.0$, where P is the pulsation period in days.

However, the non-linearity of the LMC PL relation has been proposed by Tammann & Reindl (2002), Tammann et al. (2002) and Kanbur & Ngeow (2004), i.e. the LMC data are more consistent with two PL relations and a discontinuity at a period around 10 days. This is illustrated in the lower panels of Figure 1 for the V-band LMC PL relation. The existence of two LMC PL relations is further supported by the results from a rigorous statistical test (the F-test), as presented in Kanbur & Ngeow (2004), which shows that the V- and I-band LMC PL relations are better described by two PL relations. Since the work of Tammann & Reindl (2002) and Kanbur & Ngeow (2004) are based on the OGLE (Optical Gravitational Lensing Experiment) LMC Cepheids (Udalski et al. 1999b), which are truncated at $\log(P) \sim 1.5$, Sandage et al. (2004) and Kanbur et al. (2005) used additional data that are available from the literature, especially those with $\log(P) > 1.5$, to further support the existence of two PL relations in the LMC. In Tables 1 & 2, we collect the slopes and the zero-points (ZP) for the long ($\log[P] > 1.0$) and short period PL relations that are available in the literature[1], respectively. For completeness, we also include the linear, unbroken PL relation obtained using all Cepheids

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[1] The I-band PL relations from Kanbur & Ngeow (2004) are fitted with slightly different definition of the I-band mean magnitudes. Hence we refit the I-band PL relations with the con-

Table 1. Slopes for the LMC PL relations[a]. Long and short periods are referred to $P > 10$ and $P < 10$ days, respectively.

Band	Period-Range	TR02	STR04	KN04	KNB05
V	All	-2.760 ± 0.031[b]	-2.702 ± 0.028	-2.746 ± 0.043	-2.714 ± 0.034
V	Short	-2.86 ± 0.05	-2.963 ± 0.056	-2.948 ± 0.065	-2.937 ± 0.060
V	Long	-2.48 ± 0.17	-2.567 ± 0.102	-2.350 ± 0.252	-2.563 ± 0.115
I	All	-2.962 ± 0.021[b]	-2.949 ± 0.020	-2.965 ± 0.028	-2.953 ± 0.023
I	Short	-3.03 ± 0.03	-3.099 ± 0.038	-3.096 ± 0.043	-3.090 ± 0.041
I	Long	-2.82 ± 0.13	-2.822 ± 0.084	-2.737 ± 0.179	-2.890 ± 0.081
B	All	···	-2.340 ± 0.037	···	···
B	Short	-2.42 ± 0.08	-2.683 ± 0.077	···	···
B	Long	-1.89 ± 0.62[c]	-2.151 ± 0.134	···	···

[a] The references are: TR02 = Tammann & Reindl (2002); STR04 = Sandage et al. (2004); KN04 = Kanbur & Ngeow (2004); KNB05 = Kanbur et al. (2005)

[b] Since Tammann & Reindl (2002) does not give the results from the fit to all Cepheids, we adopted the slopes from Udalski et al. (1999a) because the same dataset is used in both papers.

[c] The number of long period Cepheids in B-band is ~ 13, hence the error is larger than the others.

Table 2. Same as Table 1, but for the zero-points of the PL relations, by assuming $\mu_{LMC} = 18.50$mag.

Band	Period-Range	TR02	STR04	KN04	KNB05
V	All	-1.458 ± 0.021[a]	-1.451 ± 0.022	-1.401 ± 0.030	-1.426 ± 0.024
V	Short	-1.40 ± 0.03	-1.295 ± 0.036	-1.284 ± 0.041	-1.295 ± 0.038
V	Long	-1.75 ± 0.20	-1.594 ± 0.135	-1.795 ± 0.298	-1.564 ± 0.147
I	All	-1.942 ± 0.014[a]	-1.896 ± 0.015	-1.889 ± 0.019	-1.898 ± 0.016
I	Short	-1.90 ± 0.02	-1.806 ± 0.024	-1.813 ± 0.027	-1.817 ± 0.026
I	Long	-2.09 ± 0.15	-2.044 ± 0.111	-2.109 ± 0.212	-1.943 ± 0.104
B	All	···	-1.160 ± 0.029	···	···
B	Short	-1.18 ± 0.05	-0.955 ± 0.049	···	···
B	Long	-1.65 ± 0.74[b]	-1.364 ± 0.177	···	···

[a] Since Tammann & Reindl (2002) does not give the results from the fit to all Cepheids, we adopted the zero-points from Udalski et al. (1999a) because the same dataset is used in both papers.

[b] The number of long period Cepheids in B-band is ~ 13, hence the error is larger than the others.

in these tables. The slopes and ZPs from different papers are consistent and agree with each other. Note that these LMC PL relations have been corrected for the reddening. The reason that the LMC PL relation is non-linear is because the period-colour (PC) relation for LMC Cepheids is also non-linear across the 10 days period (Tammann et al. 2002; Kanbur & Ngeow 2004; Sandage et al. 2004; Kanbur et al. 2005). The detailed investigation of the physics behind the broken LMC PL and PC relation is beyond the scope of this paper, but it is of great interest for the studies of stellar pulsation and evolution.

The existence of two LMC PL relations suggests that in future distance scale studies, the appropriate LMC PL relation may need to be applied to the long and short period Cepheids, respectively (see, e.g., Kanbur et al. 2003). However, all of the previous applications of the LMC PL relation were based on the linear version (in a sense, it is an approximation of two PL relations). Hence an immediate question that arises is: how does the existence of two LMC PL relations affect previous studies? (A similar question was also asked by Feast 2003.) In this paper we show that if the Cepheid distance to a galaxy is derived using the Wesenheit function (or the equivalent μ_0 in equation [4]), for example in the H_0 Key Project (Freedman et al. 2001, and references therein) or in the Araucaria Project (Gieren et al. 2004; Pietrzyński et al. 2004), then the results might not be affected (see Section 2.3 & 2.4 for details). This is because the Wesenheit function is linear, even though the PL and PC relations that make up the Wesenheit function are not, as shown in Section 2.2. Given that the LMC PL relation is not linear, and the Wesenheit function has been applied in many papers (see the references in Section 2), we would like to examine the effect of broken LMC PL and PC relation on the application of the Wesenheit function to the distance scale.

ventional and reddening corrected I-band mean magnitudes. The results from the F-test remain unchanged.

2 THE WESENHEIT FUNCTION AND ITS APPLICATION IN DISTANCE SCALE

2.1 Definition

The Wesenheit function is defined as W = magnitude $-$ $R \times$ colour, where R is the ratio of total-to-selective absorption (Baraffe & Alibert 2001; Böhn-Vitense 1997; Brodie & Madore 1980; Caputo et al. 2000; Fouqué et al. 2003; Freedman 1988; Freedman et al. 1991, 2001; Gieren et al. 1998; Groenewegen 2000; Groenewegen & Oudmaijer 2000;

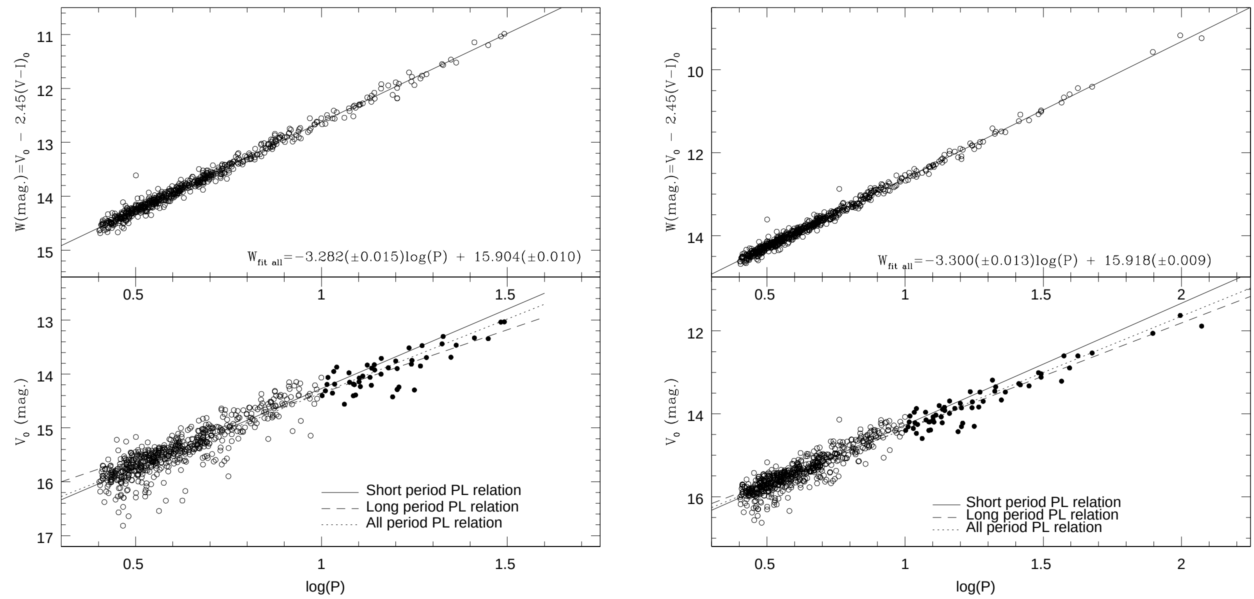

Figure 1. Comparison of the Wesenheit function with the *V*-band PL relation for the LMC Cepheids. On the top panels, we plot the Wesenheit function for the LMC data, which clearly show that the Wesenheit function is linear. We fit the data with a single line regression as indicated on the figures. On the bottom panels, we show the *V*-band PL relations, and separate the data into the short (open circles) and long period (filled circles) Cepheids. The fitted PL relations for long, short and all period Cepheids are drawn as dashed, solid and dotted lines, respectively. We extend the short period PL relation to the longer period ranges in order to compare with the long period PL relations. The data in the left and right panels are taken from Kanbur & Ngeow (2004) and Kanbur et al. (2005), respectively.

Madore 1976, 1982; Madore & Freedman 1991; Moffett & Barnes 1986; Opolski 1983; Persson et al. 2004; Storm et al. 2004; Tanvir 1997, 1999; Tanvir & Boyle 1999; Udalski et al. 1999a). A few variations of W, that are used in the literature with different combinations of magnitudes and colours, are:

$$W_V^{BV} = V - R(B-V),\ R = A_V/E(B-V), \quad (1)$$
$$W_V^{VI} = V - R(V-I),\ R = A_V/E(V-I), \quad (2)$$
$$W_I^{VI} = I - R(V-I),\ R = R_I, \quad (3)$$

where B, V & I denote the (intensity) mean magnitudes. Note that the other definition of W, as given by van den Bergh (1975), is different than the one given in equation (1). The van den Bergh version of W replaces R by the slope of the constant-period line in the colour-magnitude diagram (CMD). Madore & Freedman (1991) (also in Moffett & Barnes 1986) have pointed out some problems with the van den Bergh version and the advantage of using R in the definition of W. The biggest advantage of using W is that it is reddening-free (see, for example, Madore & Freedman 1991), i.e. $W = V - R(B-V) = V_0 - R(B-V)_0 \equiv W_0$, where V_o and $(B-V)_0$ denote the intrinsic visual magnitude and colour, as the effect of interstellar extinction on the observed magnitude and colour cancel out for a star (not only for Cepheids). Another advantage of using W is that the scatter in the W-log(P) plot is reduced (Madore 1982; Madore & Freedman 1991; Böhn-Vitense 1997; Tanvir 1997, 1999; Udalski et al. 1999a), as compared to the scatter in the *V*- or *I*-band PL relations (see Figure 1). The remaining scatter is due to the combination of photometric errors and the finite width of the instability strip (for example, see Brodie & Madore 1980; Madore & Freedman 1991; Böhn-Vitense 1997; Gieren et al. 1998). Furthermore, an equivalent definition of W with the combination of absolute magnitude and colour can be formulated, as $W_{\mathrm{Mv}} = M_V + R(M_V - M_I)$, then the distance modulus can be obtained, i.e. $\mu_W = W - W_{\mathrm{Mv}}$. It is straight forward to show that the following equation:

$$\mu_0 = \mu_V - R(\mu_V - \mu_I) \quad (4)$$

is equivalent to μ_W, where equation (4) is frequently applied in determining the extra-galactic Cepheid distances (see, for example, Allen & Shanks 2004; Freedman et al. 1991, 2001; Kanbur et al. 2003; Leonard et al. 2003; Saha et al. 1996, 2001; Tanvir 1997; Tanvir et al. 1999). Therefore, using equation (4) to obtain the distance is equivalent to obtaining the distance by fitting the W-log(P) plane with the W_{Mv}-log(P) relation.

2.2 Testing the Linearity of the Wesenheit Function

Both the Wesenheit function and equation (4) can be written as a combination of *V*- and *I*-band PL relations, and they are adopted from the LMC PL relations to derive distances. However, as we mentioned in the Introduction, the LMC PL relations are not linear, hence the applicability of the Wesenheit function and equation (4) is immediately in question. In this sub-section we would like to test the linearity of the Wesenheit function as follows.

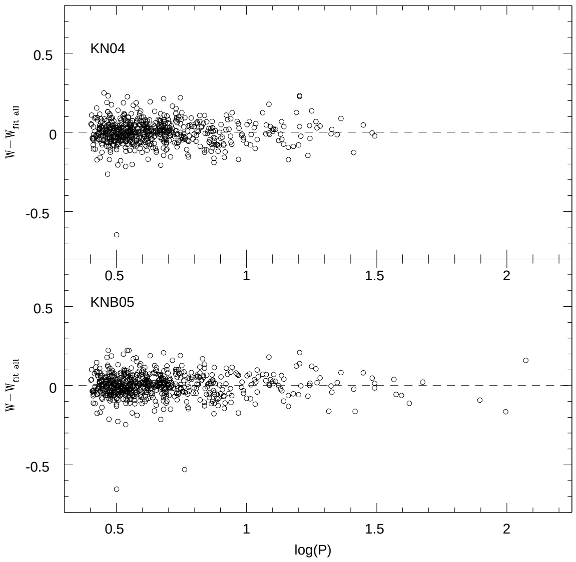

Figure 2. The residuals of W from Figure 1. The dashed line indicates the zero residuals. The residuals are scattered around the dashed lines, indicating that the Wesenheit function is linear.

The PL relation in bandpass λ can be written as: $M_\lambda = \alpha^X_\lambda + \beta^X_\lambda \log(P)$. The superscript X denotes the adopted period range, which is either for short (S, $\log[P] < 1.0$), long (L, $\log[P] > 1.0$) or all (A, short+long) periods. Then the V-$(V-I)$ Wesenheit function becomes (similar expressions can be derived for other magnitude-colour combinations):

$$W^X = (1-R)\alpha^X_V + R\alpha^X_I + [(1-R)\beta^X_V + R\beta^X_I]\log(P), \quad (5)$$

The linearity of W demands that:

$$(1-R)\beta^S_V + R\beta^S_I = (1-R)\beta^L_V + R\beta^L_I \quad \text{(for slope)}, \quad (6)$$
$$(1-R)\alpha^S_V + R\alpha^S_I = (1-R)\alpha^L_V + R\alpha^L_I \quad \text{(for ZP)}. \quad (7)$$

By using the slopes in Table 1, we can calculate the values of the LHS and RHS in equation (6), as well as the slope for W^A by using the unbroken PL relation with equation (5). The results are summarized in Table 3 with different magnitude-colour combinations. The adopted values for R in these combinations are: $R = A_V/E(B-V) = 3.24$ (Udalski et al. 1999b); $R = A_V/E(V-I) = 2.45$ (Freedman et al. 2001; Tanvir & Boyle 1999); and $R = R_I = 1.55$ (Udalski et al. 1999a). The errors in Table 3 are estimated with the standard formula for propagation of errors, i.e. $\sigma^2 = (1-R)^2\sigma^2_{\beta_V} + R^2\sigma^2_{\beta_I}$. The same is done for the zero-points in Table 4.

It can be seen immediately from Table 3 & 4 that the short period Wesenheit function agrees and is consistent with the long period Wesenheit function, as demanded by equation (6) & (7). Therefore, the Wesenheit function can be regarded as a linear function of $\log(P)$. Furthermore, the short and long period Wesenheit functions also agree and are consistent with the Wesenheit function obtained from using all Cepheids in the LMC or the linear, unbroken PL relation. Note that the value of ~ -3.3 for the slope of the Wesenheit function (β_W) with I-$(V-I)$ combination also agrees with the values given in Udalski et al. (1999a, $\beta_W = -3.277 \pm 0.014$) or in Udalski (2000, $\beta_W = -3.300 \pm 0.011$). The linearity of the Wesenheit function can be immediately seen from the top panels of Figure 1. The residuals of the W from the fitted regressions in Figure 1 are also plotted out as function of period in Figure 2. If the Wesenheit function is non-linear and can be broken into long and short period Wesenheit functions, as in the LMC PL or PC relations, then the residual plots are expected to show a trend for the long period Cepheids (as in the figure 4 from Kanbur & Ngeow 2004 for the PC relation). However, there is no obvious trend of the residuals seen from Figure 2. This further supports the linearity of the Wesenheit function.

A better and more sophisticated test of the linearity is using the F-test (Weisberg 1980; Kanbur & Ngeow 2004). The null hypothesis in our F-test is that a single linear regression is sufficient, while the alternate hypothesis is that two linear regressions are needed to describe the data. The setup and the formalism for the F-test is given in Kanbur

Table 3. Comparison of the slopes for W obtained using the Cepheids with short, long and all periods.

Dataset	Short	Long	All
	V-$(V-I)$ Combination, $R = 2.45$.		
TR02	-3.276 ± 0.103	-3.313 ± 0.403	-3.255 ± 0.068
STR04	-3.296 ± 0.124	-3.192 ± 0.253	-3.307 ± 0.064
KN04	-3.311 ± 0.141	-3.298 ± 0.571	-3.283 ± 0.093
KNB05	-3.312 ± 0.133	-3.364 ± 0.259	-3.300 ± 0.075
	I-$(V-I)$ Combination, $R = 1.55$.		
TR02	-3.293 ± 0.109	-3.347 ± 0.423	-3.275 ± 0.072
STR04	-3.310 ± 0.130	-3.217 ± 0.266	-3.332 ± 0.067
KN04	-3.325 ± 0.149	-3.337 ± 0.601	-3.304 ± 0.098
KNB05	-3.327 ± 0.140	-3.397 ± 0.273	-3.323 ± 0.079
	V-$(B-V)$ Combination, $R = 3.24$.		
TR02	-4.286 ± 0.335	-4.392 ± 2.134	$\cdots$
STR04	-3.870 ± 0.344	-3.915 ± 0.613	-3.875 ± 0.169

Table 4. Same as Table 3, but for the zero-points.

Dataset	Short	Long	All
	V-$(V-I)$ Combination, $R = 2.45$.		
TR02	-2.625 ± 0.066	-2.583 ± 0.468	-2.644 ± 0.046
STR04	-2.547 ± 0.079	-2.696 ± 0.335	-2.541 ± 0.049
KN04	-2.580 ± 0.089	-2.564 ± 0.676	-2.597 ± 0.064
KNB05	-2.574 ± 0.084	-2.493 ± 0.332	-2.582 ± 0.052
	I-$(V-I)$ Combination, $R = 1.55$.		
TR02	-2.675 ± 0.069	-2.617 ± 0.492	-2.692 ± 0.048
STR04	-2.598 ± 0.083	-2.741 ± 0.352	-2.586 ± 0.051
KN04	-2.633 ± 0.094	-2.596 ± 0.711	-2.645 ± 0.067
KNB05	-2.626 ± 0.089	-2.530 ± 0.350	-2.630 ± 0.055
	V-$(B-V)$ Combination, $R = 3.30$.		
TR02	-2.113 ± 0.206	-2.074 ± 2.543	$\cdots$
STR04	-2.397 ± 0.220	-2.339 ± 0.810	2.394 ± 0.132

& Ngeow (2004). By calculating the F value with N data points (note that F is a function of N), we can obtain the probability, $p(F)$, under the null hypothesis, for the significance of the F value. In general, the null hypothesis can be rejected if F is large (e.g., $F > 4$) or $p(F)$ is small. For example, $p(F) < 0.05$ indicates that the null hypothesis can be rejected at 2σ level. By using the 634 Cepheids as given in Kanbur & Ngeow (2004), after correcting for the extinction, we found:

$F = 1.376,\ p(F) = 0.252$ for $W_0 = V_0 - 2.45(V - I)_0$,
$F = 0.854,\ p(F) = 0.426$ for $W_0 = I_0 - 1.55(V - I)_0$.

Similarly, for the 639 Cepheids in Kanbur et al. (2005) the results from the F-test are:

$F = 1.829,\ p(F) = 0.162$ for $W_0 = V_0 - 2.45(V - I)_0$,
$F = 1.896,\ p(F) = 0.151$ for $W_0 = I_0 - 1.55(V - I)_0$.

The small values of F and the large values of $p(F)$ indicate that the null hypothesis cannot be rejected. Hence the results from the F-test strongly suggest that the Wesenheit function is linear. The results *without* the extinction corrections or the removal of the obvious outliers in upper panels of Figure 1 and in Figure 2 are essentially the same.

By combining the results obtained from this subsection, we found that, statistically, the Wesenheit function for the LMC Cepheids is indeed linear, although the LMC PL and PC relations are not. Figure 1 shows a comparison between the linear Wesenheit function and the broken V-band PL relation obtained from using two datasets. Therefore, for the LMC Cepheids, we conclude that:

Corollary A: The Wesenheit function is statistically linear, and the Wesenheit functions for short ($P <$ 10days), long and all (short+long) period Cepheids are approximately the same, i.e.,

$$W^S \sim W^L \sim W^A. \tag{8}$$

The fundamental reason that the Wesenheit function is linear is because the PC relation, e.g. $(V - I) = a + b\log(P)$, is also broken for the LMC Cepheids, in addition to the broken PL relations. The non-linearities for both of the PL and PC relations cancel out when the Wesenheit function is formulated.

2.3 Distance Scale Application

For an ensemble of Cepheids in a target galaxy, with N of them being used in determining the distance, then the distance modulus for the i^{th} Cepheid in bandpass λ (usually V and I) is $\mu^i_\lambda = m^i_\lambda - \beta_\lambda \log(P_i) - \alpha_\lambda$. The distance modulus in bandpass λ for these Cepheids can be obtained by taking the unweighted mean to individual Cepheids, i.e. $\overline{\mu_\lambda} = \frac{1}{N}\sum_{i=1}^{N}\mu^i_\lambda$. The reddening-free distance modulus for the i^{th} Cepheid can be calculated with equation (4), and it is straight forward to show that:

$$\overline{\mu_0} = \frac{1}{N}\sum_{i=1}^{N}\mu^i_0 \tag{9}$$
$$= \overline{\mu_V} - R(\overline{\mu_V} - \overline{\mu_I}), \tag{10}$$

where $\mu^i_0 = \mu^i_V - R(\mu^i_V - \mu^i_I)$. This procedure (see, for example, Freedman et al. 2001; Leonard et al. 2003; Kanbur et al. 2003; Tanvir 1997), i.e. calculating the distance modulus for individual Cepheids and taking the unweighted average to be the final distance modulus to that target galaxy (equation [9]), is equivalent to fitting the V- and I-band PL relations to the data and obtaining the μ_V and μ_I that apply to equation (4) (i.e., using equation [10]). Although both equations (9) and (10) give the same distance modulus, equation (9) assumes that $\mu_0 = \mu_V - R(\mu_V - \mu_I)$ (equation [4]) is applicable to *individual* Cepheids in a target galaxy, while equation (10) assumes that equation (4) is only applicable to an *ensemble* of Cepheids in the same target galaxy. The first approach (equation [9]) is equivalent to applying the Wesenheit function in the distance modulus determination, and since this paper is dealing with the Wesenheit function, we will use the first approach in all of the subsequent discussion.

As mentioned before, μ_0 from equation (4) is equivalent to $\mu_W = W - W_{\mathrm{Mv}}$, i.e., $\mu_0 = \mu_W$. Since the Wesenheit function (both W and W_{Mv}) for LMC Cepheids is linear, then μ_0 to a target galaxy can be obtained with the unbroken, linear version of the PL relations. Based on the corollary A (equation [8]) from the previous sub-section, we expect that the μ_0 obtained from using either the broken PL relation (for long and short period Cepheids) or the linear PL relation (for all Cepheids) should agree well with each other. Hence, when using LMC PL relations, we conclude that:

Corollary B: The distance moduli are approximately the same when using short, long and all period Cepheids, i.e.,

$$\mu^S_0 \sim \mu^L_0 \sim \mu^A_0. \tag{11}$$

Attempts to use the broken LMC PL relations to calibrate the extra-galactic Cepheid distances can be found in, e.g., Kanbur et al. (2003); Leonard et al. (2003); Thim et al. (2003, 2004). In Kanbur et al. (2003), the distance moduli to 25 *HST* observed galaxies are calculated with different LMC PL relations (see their table 5). By comparing the distance moduli derived from the U99 PL relations (linear PL relation) and the TR02 PL relation (broken long period PL relation) in their table 5, as these two PL relations are obtained using the same dataset (see Table 1), the unweighted average for the difference in distance moduli is -0.021 ± 0.002mag., in the sense that the long period broken PL relation (TR02) systematically produces slightly larger distance moduli than the linear PL relation (U99), as illustrated in Figure 3. Restricted to the 6 galaxies (NGC 925, NGC 1326A, NGC 2541, NGC 3198, NGC 3319 & IC 4182) with metallicity close to the LMC value, the difference becomes -0.017 ± 0.003mag., which is almost identical to the value when using all 25 galaxies.

In fact, the difference in the distance modulus ($\Delta\mu_0$) when using the linear and broken LMC PL relation can be calculated if the mean log-period ($< \log(P) >$) of the Cepheids in the target galaxy is known. For the distance modulus obtained with equation (4) & (9), this difference can be expressed as (Kanbur et al. 2003):

$$\Delta\mu_0 = [(R-1)\Delta\beta_V - R\Delta\beta_I] < \log(P) > +(R-1)\Delta\alpha_V - R\Delta\alpha_I, \tag{12}$$

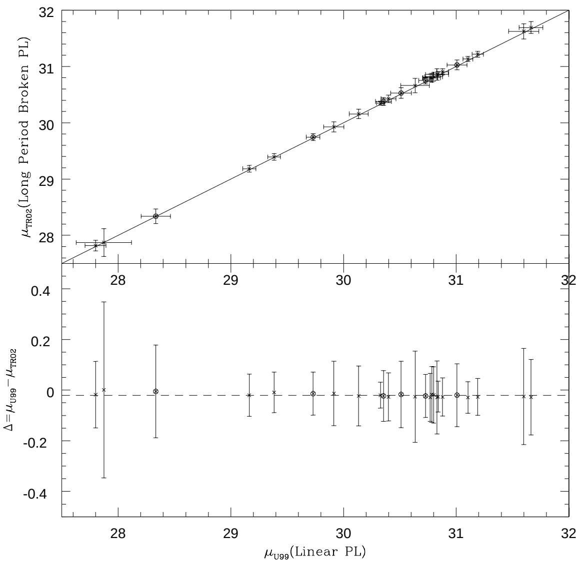

Figure 3. Comparison of the distance moduli for 25 *HST* observed galaxies adopted from Kanbur et al. (2003). The crosses with circles are the galaxies with metallicity close to the LMC value. The crosses without the circles are the remaining galaxies. The upper panel compares the distance moduli by using the linear (U99) and the long period broken (TR02) PL relations from Udalski et al. (1999a) and Tammann & Reindl (2002), respectively. The line represents the case of $\mu_{U99} = \mu_{TR02}$. The lower panel plots the difference in distance moduli ($\Delta = \mu_{U99} - \mu_{TR02}$) using these two PL relations as a function of the distances, where the dashed line represents the average difference of -0.02.

where $\Delta\beta_{(V,I)}$ and $\Delta\alpha_{(V,I)}$ are the differences in slopes and ZPs for the linear and the broken (either long or short period) PL relations. The coefficients for equation (12) are given in Table 5 for the PL relations give in Table 1 & 2. We also plot out the equation (12) in Figure 4 with the coefficients presented in Table 5. The left panel of Figure 4 suggests that the maximum value of $\Delta\mu_0$ resulting from using the linear and broken short period PL relation is ~ 0.02mag., which is small. This is not a surprise because the LMC Cepheids are dominated by the short period Cepheids (Feast 2001, 2003; Kanbur et al. 2003; Saha et al. 2001; Sandage et al. 2004; Udalski et al. 1999a), hence the linear PL relation is closer to the short period PL relation.

Since most of the target galaxies observed by *HST* are far away (> 10Mpc), usually only the long period Cepheids in a target galaxy are observed (also true for the ground-based observations to some nearby galaxies), therefore we discuss the comparison of the distance modulus obtained from the linear and broken long period PL relations in greater detail. From the right panel of Figure 4, it can be seen that $\Delta\mu_0$ is bounded within $\sim \pm0.07$mag. Therefore, the maximum difference in distance modulus between using the linear and long period broken PL relation is ~ 0.07mag. for the observed galaxies with long period Cepheids. This difference corresponds to $\sim$ 3.3% change in distance (in Mpc). The mean log-period for a target galaxy is likely to fall in the range from ~ 1.2 to ~ 1.6. Therefore, within this range, the maximum value that $\Delta\mu_0$ can have is about ±0.03mag. For the 25 *HST* observed galaxies, the average

Table 5. The coefficients to calculate the change of distance modulus ($\Delta\mu_0$) when using the linear and long/short period broken PL relation with different datasets[a].

Dataset	c	d	$< \log(P) >_0$
$\Delta\mu_0$ =Linear PL − Long Period Broken PL			
TR02	-0.0581	0.0608	1.046
STR04	0.1154	-0.1553	1.345
KN04	-0.0156	0.0323	2.071
KNB04	-0.0646	0.0899	1.391
$\Delta\mu_0$ =Linear PL − Short Period Broken PL			
TR02	-0.0216	0.0188	0.870
STR04	0.0110	-0.0057	0.521
KN04	-0.0281	0.0166	0.590
KNB04	-0.0123	0.0085	0.691

[a] $\Delta\mu_0 = c < \log(P) > +d$ from equation (12) with $R = 2.45$. $< \log(P) >_0$ is the mean log-period such that $\Delta\mu_0 = 0$.

of the $< \log(P) >$ is ~ 1.4 (Kanbur et al. 2003), hence the values of $\Delta\mu_0$ by using the STR04 and KNB05 PL relations is smaller than 0.01mag., which is negligible. The other two sets of PL relations (TR02 & KN04) predict a slightly larger $\Delta\mu_0$ of $\sim \pm0.02$mag. Note that the TR02 PL relations give larger distance modulus when using the long period broken PL relation rather than the linear PL relation for $< \log(P) >$ greater than ~ 1. In contrast, the KN04 PL relations will give a smaller distance modulus with the long period broken PL relation almost in all galaxies (unless $< \log(P) >$ is greater than 2, which is very unlikely).

In short, the maximum *systematic* error in distance modulus induced when using the linear or broken LMC PL relations is $\sim$ 0.07mag. (and even smaller if one uses the short period PL relation), with typical value around $\sim$ 0.03mag. This result justifies the corollary we have from equation (11), and both of the linear and the broken LMC PL relations can be applied to obtain the Cepheid distances.

2.3.1 A Note on the Random Errors for μ_0 from Equation (4)

By definition, the random error for $\overline{\mu} = \frac{1}{N}\sum_{i=1}^{N}\mu^i$, due to purely statistical fluctuations, is:

$$\sigma^2 = \frac{1}{N(N-1)}\sum_{i=1}^{N}(\mu^i - \overline{\mu})^2. \quad (13)$$

This equation holds for the $\overline{\mu_\lambda}$ and $\overline{\mu_0}$ from equation (9). We can expand the expression for σ_0^2 by substituting the expression for μ_0^i and $\overline{\mu_0}$, then:

$$\begin{aligned}\sigma_0^2 &= \frac{1}{N(N-1)}\sum_{i=1}^{N}(\mu_0^i - \overline{\mu_0})^2 \\ &= (1-R)^2\sigma_V^2 + R^2\sigma_I^2 - CORR_{VI}, \end{aligned} \quad (14)$$

where σ_V^2 and σ_I^2 are given by equation (13), and $\mathrm{CORR}_{VI} = \frac{2(R-1)R}{N(N-1)}\sum_{i=1}^{N}(\mu_V^i - \overline{\mu_V})(\mu_I^i - \overline{\mu_I})$, a term for the correlated residuals from both bands (Tanvir 1997; Freedman et al.

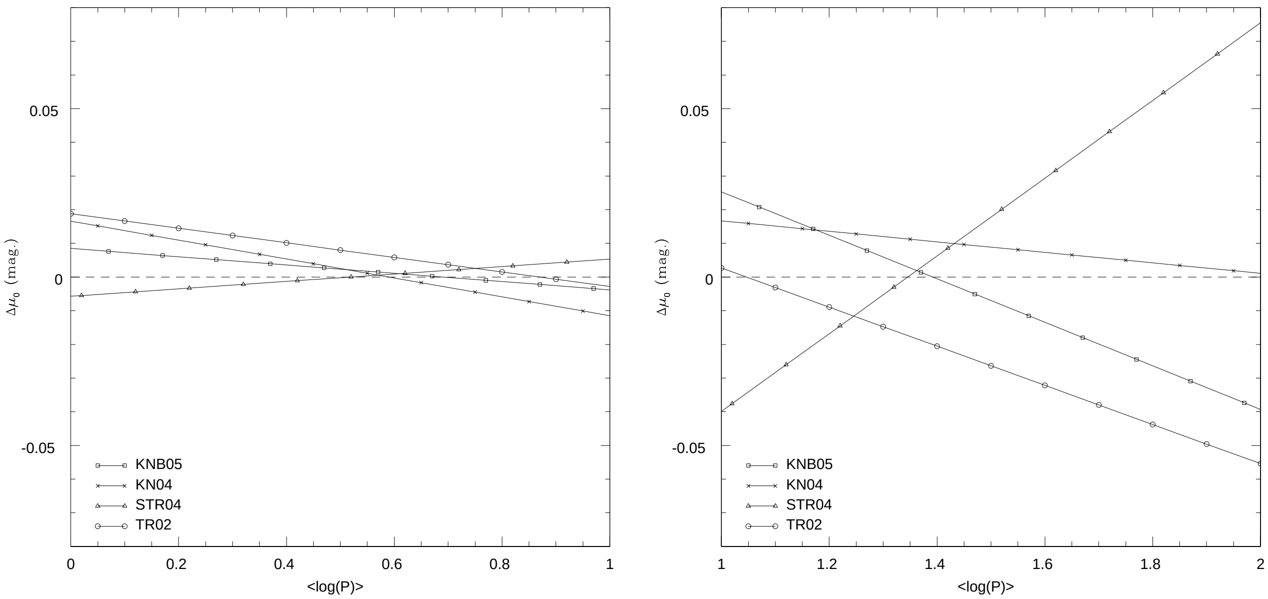

Figure 4. The difference in the distance modulus ($\Delta\mu_0$), when using the linear and broken LMC PL relations, as function of mean log-period ($< \log(P) >$). The lines are plotted with the coefficients given in Table 5. The PL relations with different datasets are given in the lower-left corner. The left and right panels show the plots of $\Delta\mu_0$ when using the broken short and long period PL relations, respectively, as compared to the linear PL relation.

2001). Note that the random error given in equation (14) is for the distance modulus μ_0 obtained from equation (9). On the other hand, if we apply the standard equation for the propagation of errors (POE) to equation (4) or (10), by ignoring the correlation term, then we have $\sigma_0^2(POE) = (1-R)^2\sigma_V^2+R^2\sigma_I^2$. By comparing this expression to equation (14), we obtain:

$$\sigma_0^2(POE) = \sigma_0^2 + \mathrm{CORR}_{VI}. \quad (15)$$

One can immediately see that $\sigma_0^2(POE)$ is always greater than σ_0^2 since the term CORR_{VI} is always positive (for $R > 1$). Errors estimated from error propagation, i.e. using equation (10), will ignore the $CORR_{VI}$ term. For example, using the linear PL relation from TR02 with the 28 Cepheids in IC 4182 (see Section 2.4), $\sigma_0^2 = 0.0028$ and $\mathrm{CORR}_{VI} = 0.0107$. The sum of these two terms is 0.0135, which is equal to $\sigma_0^2(POE) = 0.0135$. A similar calculation to NGC 4258 shows the same result. Note that all the errors discussed here, the σ, are random errors only, which do not include the systematic errors such as the errors arise from the calibrations, the width of the instability strip, and others (see, e.g., the discussion by Saha et al. 2000).

2.4 An Example: Cepheid Distance to IC 4182

It is desired to test the corollary from equation (11) to a galaxy that contains roughly equal numbers of long and short period Cepheids. In addition, this target galaxy should have metallicity close to the LMC value, to avoid any possible metallicity dependency on PL relations (or on the Wesenheit function, see Section 3.1) and to further test the applicability of the broken LMC PL relations in the distance scale. A nearby Sdm galaxy, IC 4182, provides such an opportunity, because:

(i) The metallicity, defined as $12 + \log[O/H]$, for IC 4182 is $8.40 \pm 0.20dex$, which is close to the LMC value of $8.50 \pm 0.08dex$ (see the reference in Ferrarese et al. 2000).

(ii) The number of Cepheids discovered in this galaxy is roughly equally divided into long and short periods, as the distance to this galaxy is about 4.5Mpc (Freedman et al. 2001). Most of the other *HST* observed galaxies, especially those with LMC metallicity, are further away and hence only contain the long period Cepheids.

The Cepheids in IC 4182 were discovered with *HST-WFPC* observations in 1992 (Saha et al. 1994). The photometric data from the observations have been reduced with DoPHOT (Saha et al. 1994) and ALLFRAME (Gibson et al. 2000) reduction packages. The Cepheids discovered from these packages are: 27 from Saha et al. (1994) and 28 from Gibson et al. (2000), where 27 of them are common to the Saha et al. (1994) results. We use the latest photometric results from Gibson et al. (2000) for our study in this paper. An additional advantage of using the Cepheids in this galaxy is that the extinction toward this galaxy is expected to be small (see, e.g., Saha et al. 1994; Shanks 1997; Ferrarese et al. 2000; Gibson et al. 2000; Freedman et al. 2001; Macri et al. 2001), because IC 4182 is located at high Galactic latitude ($l = 108^o$, b$= +79^o$) and is nearly face-on (Saha et al. 1994). Unfortunately, the *I*-band photometry of the Cepheids in IC 4182 only contains two epochs, in contrast to the 20 epochs of *V*-band photometry (Saha et al. 1994), therefore the measurement of the mean *I*-band magnitudes is not as accurate as the *V*-band. Better observations to IC 4182 are desired in the future to obtain more accurate *I*-

Table 6. The distance modulus (μ_0) to IC 4182[a].

LMC PL relation (1)	μ_0^S ($N=13$) (2)	μ_0^L ($N=15$) (3)	μ_0^A ($N=28$) (4)	$\mu_0(\mathrm{ave})=\frac{1}{2}(\mu_0^L+\mu_0^S)$ (5)
TR02	28.262 ± 0.088	28.290 ± 0.067	28.267 ± 0.053	28.276 ± 0.055
SRT04	28.199 ± 0.087	28.232 ± 0.065	28.222 ± 0.053	28.216 ± 0.054
KN04	28.244 ± 0.087	28.250 ± 0.067	28.250 ± 0.053	28.247 ± 0.055
KNB05	28.238 ± 0.087	28.271 ± 0.069	28.255 ± 0.053	28.255 ± 0.056

[a] The number in parentheses is the number of Cepheids used to derive the distance modulus. The superscripts ^{S}, ^{L} and ^{A} refer to the short, long and all period Cepheids, respectively.

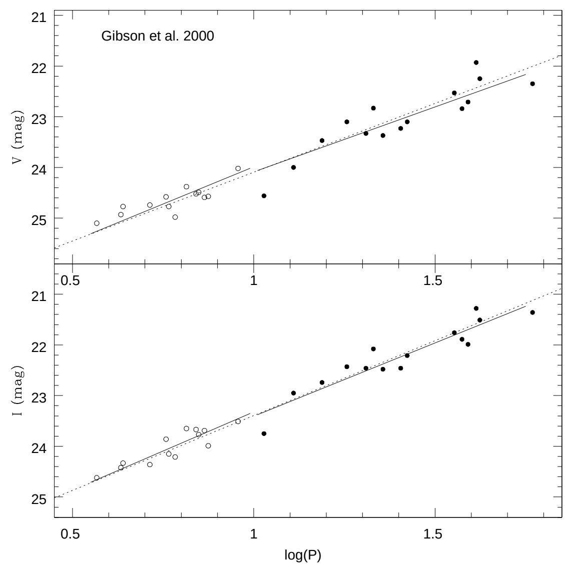

Figure 5. The PL relation for Cepheids in IC 4182 with the data taken from Gibson et al. (2000). The filled and open circles are for long and short period Cepheids, respectively. The solid and the dotted lines are the broken and linear PL relations used to fit the data, respectively. The PL relations used are taken from KNB05 in Table 1 & 2.

band light curves and increase the number of Cepheids in this galaxy.

The distance moduli to IC 4182 are obtained with equation (4) for using the short period (13 Cepheids), long period (15 Cepheids) and all Cepheids in the galaxy. The results are tabulated in Table 6 (column 2, 3 & 4) with the PL relations listed in Table 1 & 2, with the *random* errors calculated from equation (14). The fitted PL relations to the Cepheids in IC 4182 are given in Figure 5, using the KNB05 PL relations as an illustration (other sets of PL relations look similar to this figure). As can be seen from Table 6, the distance moduli in column 2, 3 & 4 agree well and are consistent to each others. Therefore, the results from Table 6 strongly support the corollary from equation (11). To reduce the random error, the distance modulus could be obtained from combining the short and long period Cepheids to increase the sample size. Then the linear PL relation can be applied because of the linearity of Wesenheit function, as well as the corollary from equation (11).

In case we want to obtain the distance modulus with the broken PL relations (as in μ_0^S and μ_0^L from Table 6), we can take the average of the long and short period distance moduli as the final adopted distance modulus to IC 4182, because the long and short period Cepheids can be regarded as two independent samples. This is done in column 5 of Table 6, which show that the averaged distance modulus (μ_0[ave]) is almost identical to the μ_0^A, which is obtained from using all Cepheids and the linear PL relations[2].

The results found in this subsection apply to only one galaxy. Hence, more galaxies with metallicity close to LMC values and containing large number of short and long period Cepheids are clearly desired in the future studies to further quantify the validity of corollary B or equation (11).

3 CONCLUSION AND DISCUSSION

Due to the recent discovery of the non-linearity for the LMC PL and PC relations, Feast (2003) has asked the following critical question:

> "$\cdots$ is there a significant slope difference between short and long ($>\sim$ 10days) Cepheids that would seriously affect the calibration and use of PL [and PC] relation?"

From this study, we showed that this problem can be remedied with the application of the Wesenheit function in distance scale studies. This is because the Wesenheit function for the LMC Cepheids is linear, as shown in this paper, although the LMC PL and PC relations are not. Therefore, the Cepheid distances obtained with the Wesenheit function or the equivalent μ_0 would not be affected with the recent finding of the broken PL and PC relations. We also found that the maximum systematic error induced from using the linear or the broken PL relations is about 0.07mag., with typical value around 0.03mag. Hence, researchers can choose to apply either the linear or the broken LMC PL relations to obtain the Cepheid distances, without worrying that these two approaches will give inconsistent results. These conclusions are further supported by the distances to IC 4182 when using different sets of LMC PL relations (long vs. short and the linear, unbroken PL relation), which shows that the distance moduli obtained when using the linear and broken LMC PL relations agree well to each other. Therefore, both approaches are equally applicable in deriving the Cepheid distance. Hence, the question raised by Feast (2003) is essentially answered. However, there is another problem

[2] This can be seen immediately if the figures in column 4 & 5 of Table 6 are rounded to 2 significant figures.

associated with the metallicity dependency (or the universality) of the PL relations and the Wesenheit function that may also affect the use and calibration of PL relations. We briefly discuss this problem in the next sub-section.

3.1 The Metallicity Dependency of the Wesenheit Function

There are some recent studies suggesting that the PL relation is not universal, i.e., the Galactic PL relation is steeper than the LMC counterparts (Tammann et al. 2003; Fouqué et al. 2003; Kanbur et al. 2003; Ngeow & Kanbur 2004; Storm et al. 2004). Recall that the study of the linearity of the Wesenheit Function in this paper is only applicable to the LMC Cepheids. Therefore, it is possible that the Wesenheit function is not universal and depends on metallicity (see, e.g., Moffett & Barnes 1986). In fact, the Wesenheit function fitted from 31 Galactic Cepheids in Storm et al. (2004) shows that the slope is steeper than the slope for the Cepheids in Small Magellanic Cloud (SMC). Their value of the slope for the V-$(V-I)$ Wesenheit function is -3.63 ± 0.11, which is also steeper than the values given in Table 3 (or Figure 1) for the LMC Cepheids[3]. On the other hand, the studies from the Araucaria Project suggests that the Wesenheit function is universal, at least for low metallicity systems (Pietrzyński et al. 2004). The theoretical study by Caputo et al. (2000) suggest that the slopes for the I-$(V-I)$ Wesenheit function are consistent for $Z = 0.004$ (SMC) and $Z = 0.008$ (LMC), with -3.21 ± 0.05 and -3.17 ± 0.04, respectively. However, the slope of the Galactic ($Z = 0.02$) I-$(V-I)$ Wesenheit function, -3.02 ± 0.04, is shallower than the slopes for the LMC and SMC models. In contrast, Baraffe & Alibert (2001) found that the theoretical slopes for I-$(V-I)$ Wesenheit function are around ~ -3.4 with these three values of Z. Therefore, the metallicity dependency of the Wesenheit function is still an open question. The detailed study of the metallicity dependency of the Wesenheit function is beyond the scope of this paper, which is subjected for a future study.

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REFERENCES

Allen, P. & Shanks, T., 2004, MNRAS, 347, 1011
Baraffe, I. & Alibert, Y., 2001, A&A, 371, 592
Böhn-Vitense, E., 1997, AJ, 113, 13
Brodie, J. & Madore, B., 1980, MNRAS, 191, 841
Caputo, F., Marconi, M. & Musella, I., 2000, A&A, 354, 610
Feast, M., 2001, Odessa Astronomical Pub., 14, 144 (astro-ph/0110360)
Feast, M., 2003, in *Stellar Candles for Extra-Galactic Distance Scale*, ed. D. Alloin & W. Gieren, Lecture Notes in Physics Vol. 635, Springer, New York, pg. 45
Ferrarese, L., Ford, H., Huchra, J., Kennicutt, R., Mould, J., Sakai, S., Freedman, W., Stetson, P., et al., 2000, ApJS, 128, 431
Fouqué, P., Storm, J. & Gieren, W., 2003, in *Stellar Candles for Extra-Galactic Distance Scale*, ed. D. Alloin & W. Gieren, Lecture Notes in Physics Vol. 635, Springer, New York, pg. 21
Freedman, W., 1988, ApJ, 326, 691
Freedman, W., Wilson, C. & Madore, B., 1991, ApJ, 372, 455
Freedman, W., Madore, B., Gibson, B., Ferrarese, L, Kelson, D., Sakai, S., Mould, J., Kennicutt, R., et al., 2001, ApJ, 553, 47
Gibson, B., Stetson, P., Freedman, W., Mould, J., Kennicutt, R., Huchra, J., Sakai, S., Graham, J., et al., 2000, ApJ, 529, 723
Gieren, W., Fouqué, P. & Gómez, M., 1998, ApJ, 496, 17
Gieren, W., Pietrzyński, G., Walker, A., Bresolin, F., Minniti, D., Kudritzki, R.-P., Udalski, A., Soszyński, I., et al., 2004, AJ, 128, 1167
Groenewegen, M. A. T., 2000, A&A, 363, 901
Groenewegen, M. A. T. & Oudmaijer, R., 2000, A&A, 356, 849
Leonard, D., Kanbur, S., Ngeow, C. & Tanvir, N., 2003, ApJ, 594, 247
Kanbur, S., Ngeow, C., Nikolaev, S., Tanvir, N. & Hendry, M., 2003, A&A, 411, 361
Kanbur, S. & Ngeow, C., 2004, MNRAS, 350, 962
Kanbur, S., Ngeow, C. & Buchler, R., 2005, in-preparation
Madore, B., 1976, Royal Greenwich Obs. Bull. 182, 153
Madore, B., 1982, ApJ, 253, 575
Madore, B. & Freedman, W., 1991, PASP, 103, 933
Macri, L., Calzetti, D., Freedman, W., Gibson, B., Graham, J., Huchra, J., Hughes, S., Madore, B., et al., 2001, ApJ, 549, 721
Moffett, T. & Barnes, T., 1986, ApJ, 304, 607
Ngeow, C. & Kanbur, S., 2004, MNRAS, 349, 1130
Opolski, A., 1983, IBVS, 2425, 1
Persson, S., Madore, B., Krzemiński, W., Freedman., W., Roth, M. & Murphy, D., 2004, AJ, 128, 2239
Pietrzyński, G., Gieren, W., Udalski, A., Bresolin, F., Kudritzki, R.-P., Soszyński, I., Szymański, M. & Kubiak, M., 2004, AJ, 128, 2815
Saha, A., Labhardt, L., Schwengeler, H., Macchetto, F., Panagia, N., Sandage, A. & Tammann, G., 1994, ApJ, 425, 14
Saha, A., Sandage, A., Labhardt, L., Tammann, G., Macchetto, F. & Panagia, N., 1996, ApJ, 466, 55
Saha, A., Labhardt, L. & Prosser, C., 2000, PASP, 112, 163
Saha, A., Sandage, A., Tammann, G., Dolphin, A., Christensen, J., Panagia, N. & Macchetto, F., 2001, ApJ, 562, 314
Sandage, A., Tammann, G. A. & Reindl, B., 2004, A&A, 424, 43
Shanks, T., 1997, MNRAS, 290, L77
Storm, J., Carney, B., Gieren, W., Fouqué, P., Latham, D., Fry, A., 2004, A&A, 415, 531
Tammann, G. A. & Reindl, B., 2002, Ap&SS, 280, 165
Tammann, G. A., Reindl, B., Thim, F., Saha, A. &

[3] Note that Storm et al. (2004) use $R = 2.51$ instead of $R = 2.45$ as we adopted here. However, the small difference in R does not fully account the difference in the slopes of Wesenheit function.

Sandage, A., 2002, in *A New Era in Cosmology*, eds. N. Metcalfe & T. Shanks, ASP Conf. Series Vol. 283, Astron. Soc. Pac., San Francisco, pg. 258

Tammann, G. A., Sandage, A. & Reindl, B., 2003, A&A, 404, 423

Tanvir, N., 1997, in *The Extra-galactic Distance Scale*, Eds. Livio, M, Donahue, M. & Panagia, N., Cambridge University Press, Cambridge, pg. 91

Tanvir, N., 1999, in *Harmonizing Cosmic Distance Scales in a Post-HIPPARCOS Era*, eds. Egret, D. & Heck, A., ASP Conf. Series Vol. 167, Astron. Soc. Pac., San Francisco, pg. 84

Tanvir, N. & Boyle, A., 1999, MNRAS, 304, 957

Tanvir, N., Ferguson, H. & Shanks, T., 1999, MNRAS, 310, 175

Thim, F., Tammann, G., Saha., A., Dolphin, A., Sandage, A., Tolstoy, E. & Labhardt, L., 2003, ApJ, 590, 256

Thim, F., Hoessel, J., Saha, A., Claver, J., Dolphin, A. & Tammann, G., 2004, AJ, 127, 2322

Udalski, A., Szymański, M., Kubiak, M., Pietrzyński, G., Soszyński, I., Wozniak, P. & Zebrun, K, 1999a, AcA, 49, 201

Udalski, A., Soszyński, I., Szymański, M., Kubiak, M., Pietrzyński, G., Wozniak, P. & Zebrun, K., 1999b, AcA, 49, 223

Udalski, A., 2000, AcA, 50, 279

van den Bergh, S., 1975, Stars & Stellar System Vol. IX, *Galaxies and the Universe*, Eds. Sandage, A., Sandage, M. & Kristian, J., University of Chicago Press, Chicago, pg. 509

Weisberg, S., 1980, *Applied Linear Regression*, John Wiley & Sons, 1^{st} Ed.