SOLUTIONS

1 Why is the temperature of a star’s core so important in determining which nuclear reactions can occur there?

Fusion reactions occur when the strong force, the force that binds the neutrons and protons, binds two nuclei in two initially separate atoms. The strong force acts at very short distances. However, the nuclei are positively charged and therefore repel at the average distances between the nuclei in the gas. Therefore, in order for binding to occur, the nuclei have to be pushed close together, overcoming the repulsion of the electric force. This requires that the nuclei have very high relative velocity.

Recall that the temperature of a gas reflects the average speed of the particles. Only the fastest particles at any given temperature will be able overcome the repulsion. Increasing the temperature will then increase the speed of the fastest particles and in turn increase the rate of fusion reactions. If the fusion reactions are too numerous, the excess heat will cause excess pressure and the core will expand, decreasing the temperature and the reaction rate. We say the system is self-regulating. This self-regulation explains the stability of a star on the main sequence.

2 The mass of a neutron is about $1.7 \times 10^{-27}$ km and its radius is about $10^{-15}$ m (also known as a femtometer). Compare the density of matter in a neutron with the density of matter in a neutron star. [Hint: a ton is about 1000 kg and a teaspoon is about a cubic cm.]

The density of neutron is:

$$m \frac{V}{4/3\pi r^3} = \frac{m}{4/3\pi(10^{-15})^3} k g/m^3 = 4.06 \times 10^{17} k g/m^3$$

The density of matter in a neutron star is:

$$\frac{1.4M_\odot}{4/3\pi R^3} = \frac{1.4 \times 1.99 \times 10^{30} k g}{4/3\pi(10^4 m)^3} k g/m^3 = 6.7 \times 10^{17} k g/m^3$$

The density of matter in a neutron and the density of matter in a neutron star are similar. The density of the neutron star is higher because the gravitational pull can compress the nuclear fluid somewhat even though it is degenerate matter.
Find the average density of a $1M_\odot$ white dwarf having the same size as the Earth. What speed is required to eject the gas from the white dwarf’s surface? Suppose some interstellar gas fell on the white dwarf. With what speed would it strike the star’s surface? [Hint: Think about escape velocity…]

The radius of Earth is about 6378 km, thus the density of this white dwarf is:

$$\frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{2.0 \times 10^{30}}{4/3\pi(6.378 \times 10^6)^3} \text{kg/m}^3 = 1.83 \times 10^8 \text{kg/m}^3.$$  

Note that this is 9 orders of magnitude smaller than the density of the neutron star.

The escape velocity is:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G M_\odot}{6378 \text{km}}} = 6.45 \times 10^6 \text{m/s} = 6450 \text{km/s}.$$  

As a sanity check, we can compute the escape velocity from the Earth:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G \times 6.0 \times 10^{24} \text{kg}}{6378 \text{km}}} = 1.12 \times 10^4 \text{m/s} = 11.2 \text{km/s}.$$  

Clearly, it would be much more difficult for a rocket to get off the surface of a white dwarf than off the Earth.

If one launches a projectile from a body with escape velocity it will have zero velocity at large distance. If one lets the projectile fall back in the to body from far away with initially zero velocity, it will have escape velocity when it reaches the surface.

Especially for the physics students, note that total energy (kinetic plus potential) at launch must be the same as the total energy far away, that is at large radius. Therefore, the kinetic energy at launch must equal the energy at escape. Far away from the body, the energy will be zero. This is how we derived escape velocity:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

where $R$ is the radius of the body. Since the energy of the escaped projectile will be zero, if it falls back to the surface of the body, the above equation must also hold. The velocity at the surface is then $v_{\text{esc}}$.

Selena flies past Michael in her spaceship at nearly the speed of light. According to Michael, Selena’s clock runs slow. According to Selena, does Michael’s clock run slow, fast or at the normal rate? Explain. [Problem 3, p. 551]

According to Selena, Michael’s clock runs slow as well. According to Selena, it is Michael who is flying by the speed of light. In her relative frame of reference, she is stationary, her clock is set to her standard time, and time runs normally. In other words, it is Michael’s clock that runs slow in her relative frame of reference because of time dilation.
If the Sun suddenly became a black hole, how would the Earth’s orbit be affected? Explain. [Problem 20, p. 551]

The Earth’s orbit would remain unchanged. According to Newton’s law of gravity, the gravitational pull on the earth $F = \frac{GM_1M_2}{r^2}$, independent of the radius of the star. In other words, because the mass of the Sun stays the same as it becomes a black hole, the gravitational force stays the same and therefore the orbit also stays the same.